AGA KHAN UNIVERSITY EXAMINATION BOARD

Notes from E-Marking Centre HSSC-I Business Mathematics Annual Examinations 2023

Introduction

This document has been prepared for the teachers and candidates of Higher Secondary School Certificate (HSSC) Part I (Class XI) Business Mathematics. It contains comments on candidates' responses to the 2023 HSSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that requires candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations

Candidates performed really well in some concepts, such as, compound interest, annuities and sum of geometric series. However, candidates who did not score well mostly failed to understand the demands of the questions, often misinterpreting the command words and the stimuli. Furthermore, interpretation of graphs and scientific reasoning were also weak.

Mentioned below are few concepts that teachers need to focus so that the candidates may perform better.

- Ratios and Proportionality
- Linear Inequalities and Simultaneous Equations
- Matrix and its Multiplication.

Note: Candidates' responses shown in this report have not been corrected for grammar, spelling, format, or information.

DETAILED COMMENTS
Constructed Response Questions (CRQs)

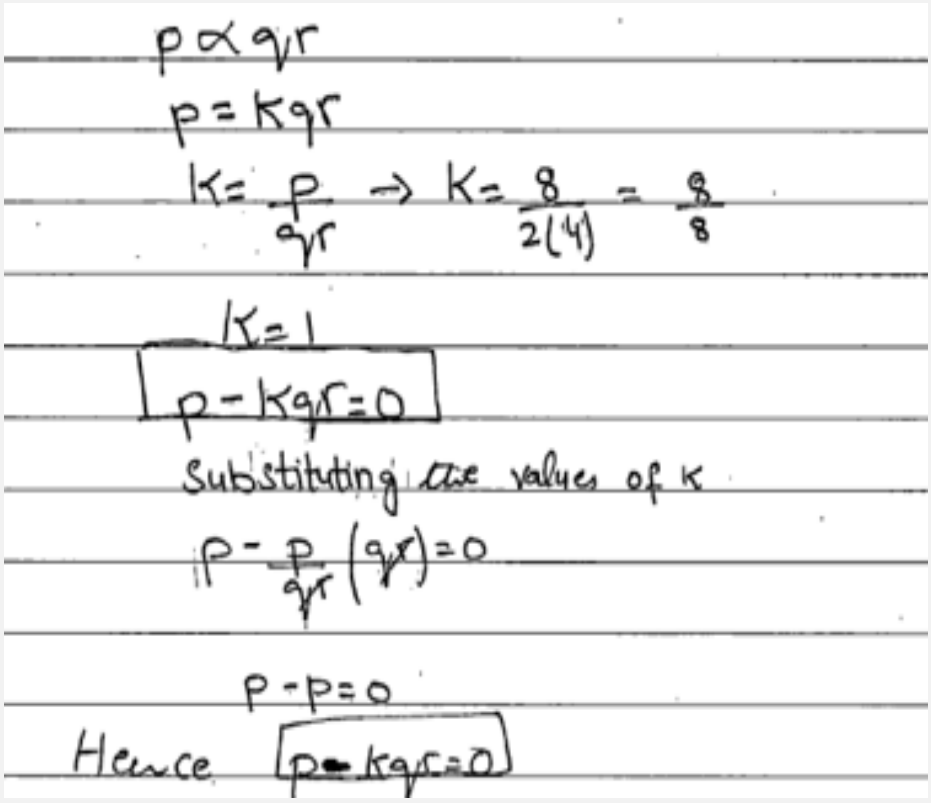
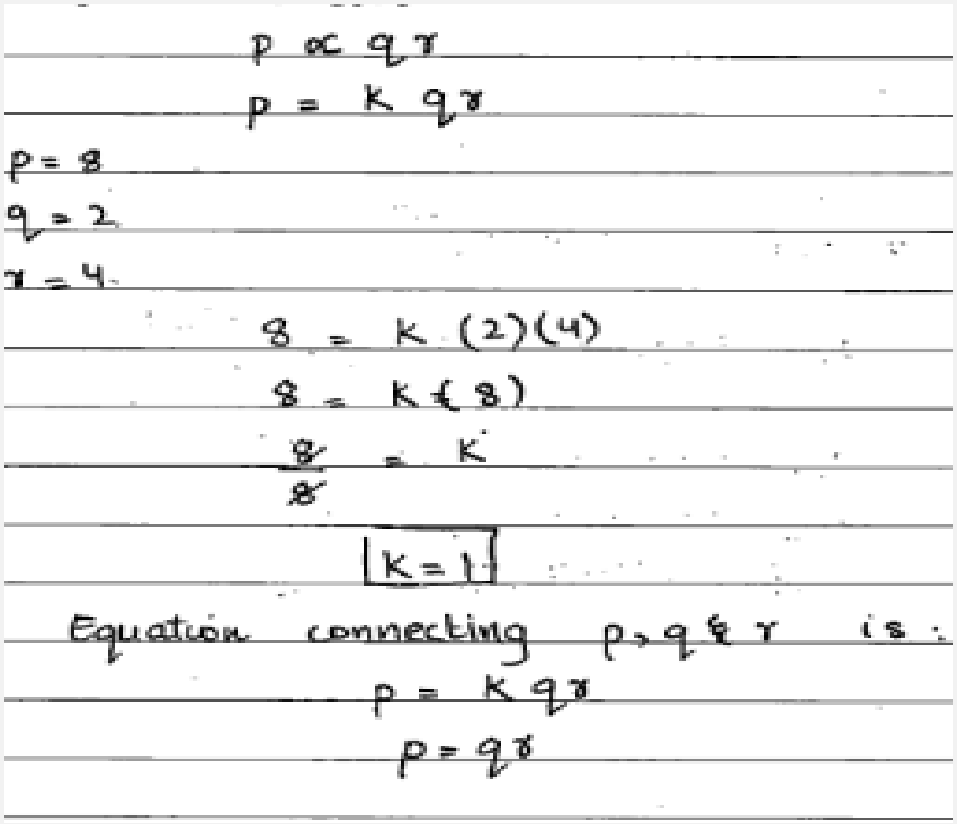
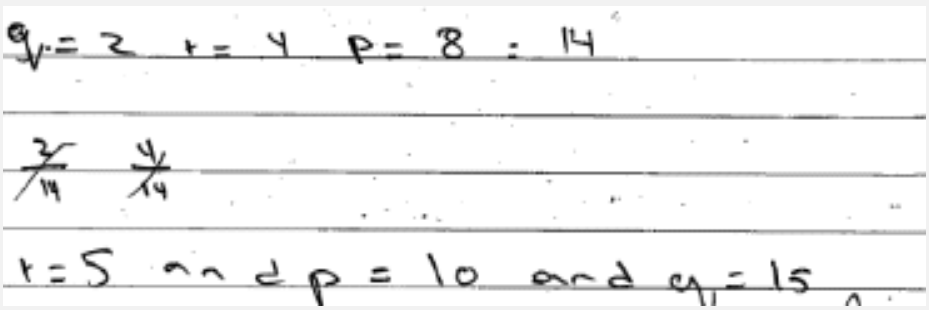
Question No. 1	
Question Text	A quantity *p* varies directly with quantity *q* and *r*. For $q = 2$ and $r = 4$, $p = 8$. Find the i. equation connecting *p*, *q* and *r*. ii. value of *q*, when the values of $r = 5$ and $p = 10$.
SLO No.	1.2.2
SLO Text	Solve word problems related to SLO 1.2.1. (direct, inverse and compound (joint and combined) variations).
Max Marks	4
Cognitive Level	A*
Checking Hints	i. 1 mark for converting proportionality into equation 1 mark for finding the value of *k*. 1 mark to find the equation. ii. 1 mark substituting the values in the equation.
Overall Performance	This question was related on direct proportion. Only few of the candidates accurately grasped the concept, recognising that when one variable increases, the others increase as well. They also correctly calculated the proportionality constant.
Description of Better Responses	In better responses, candidates correctly interpreted the question, demonstrating a clear understanding of the concept of proportionality and the proportionality constant. They accurately substituted the values and further verified their answers by confirming the value of the constant, even though it was not explicitly required in the question.
Images of Better Responses	**Part (i) Image (i)** 

	Image (ii)  $p \propto qr$ $p = kqr$ $p = 8$ $q = 2$ $r = 4$ $8 = k(2)(4)$ $8 = k(8)$ $\frac{8}{8} = k$ $k = 1$ Equation connecting p, q & r is: $p = kqr$ $p = qr$ **Part (ii)** $q = ?\ r = 5\ p = 10$ $p - kqr = 0$ $kqr = p$ $q = \frac{p}{kr}$ $q = \frac{10}{5}$ $q = 2$
Description of Weaker Responses	One common error among weaker responses was the inappropriate application of the quadratic formula. Instead of recognising the problem as one involving proportionality, these candidates attempted to solve it using the quadratic formula. This misinterpretation of the question demonstrated a lack of comprehension of the fundamental concept of proportionality and an inability to discern the appropriate mathematical approach for such problems.
Images of Weaker Responses	**Image (i)**  $q = 2\ r = 4\ p = 8 : 14$ $\frac{2}{14}\ \frac{4}{14}$ $r = 5$ and $p = 10$ and $q = 15$

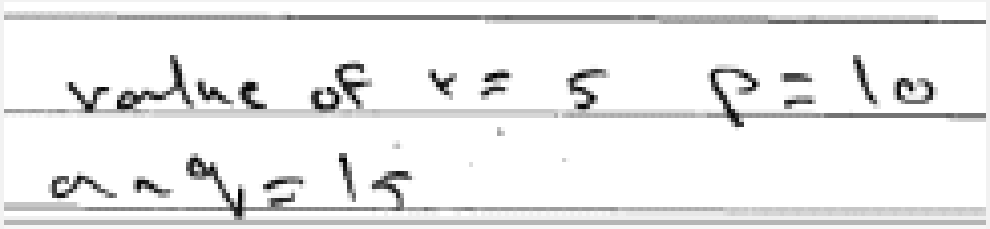
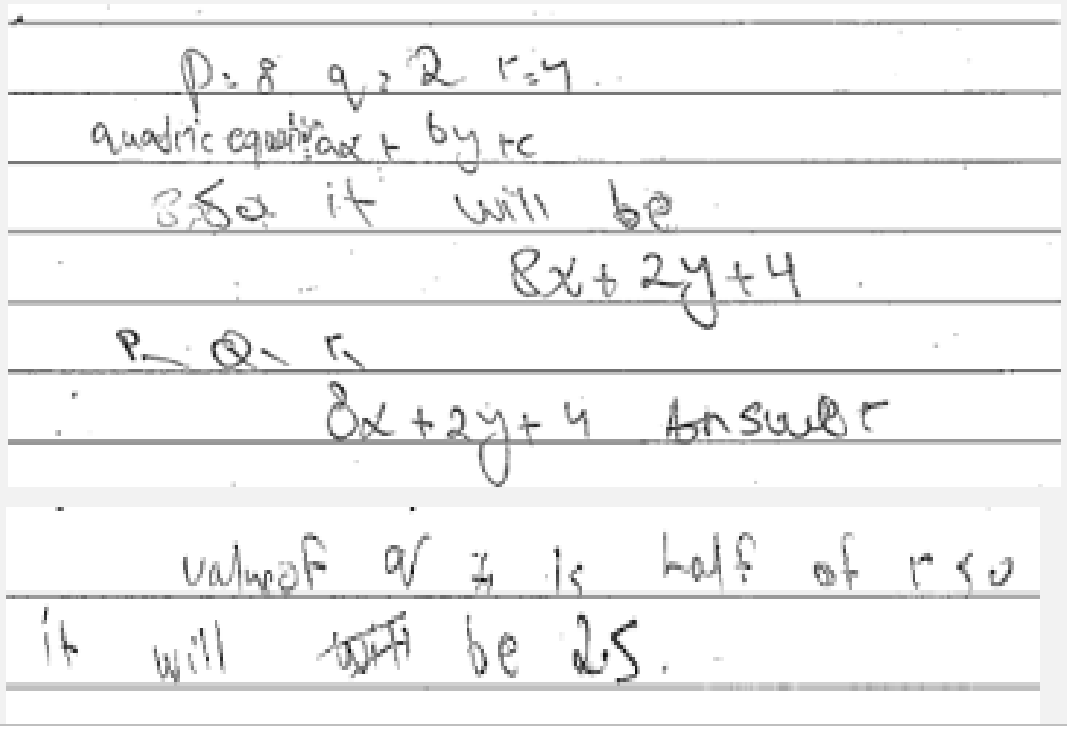


Image (ii)



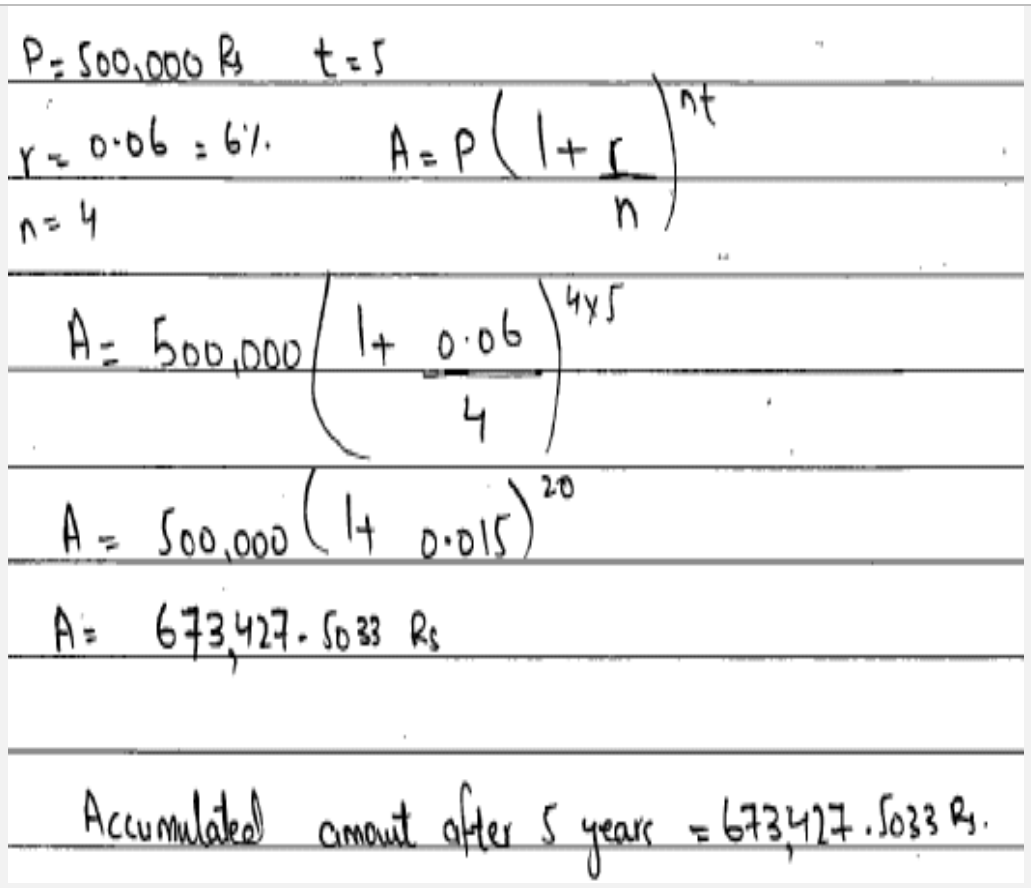
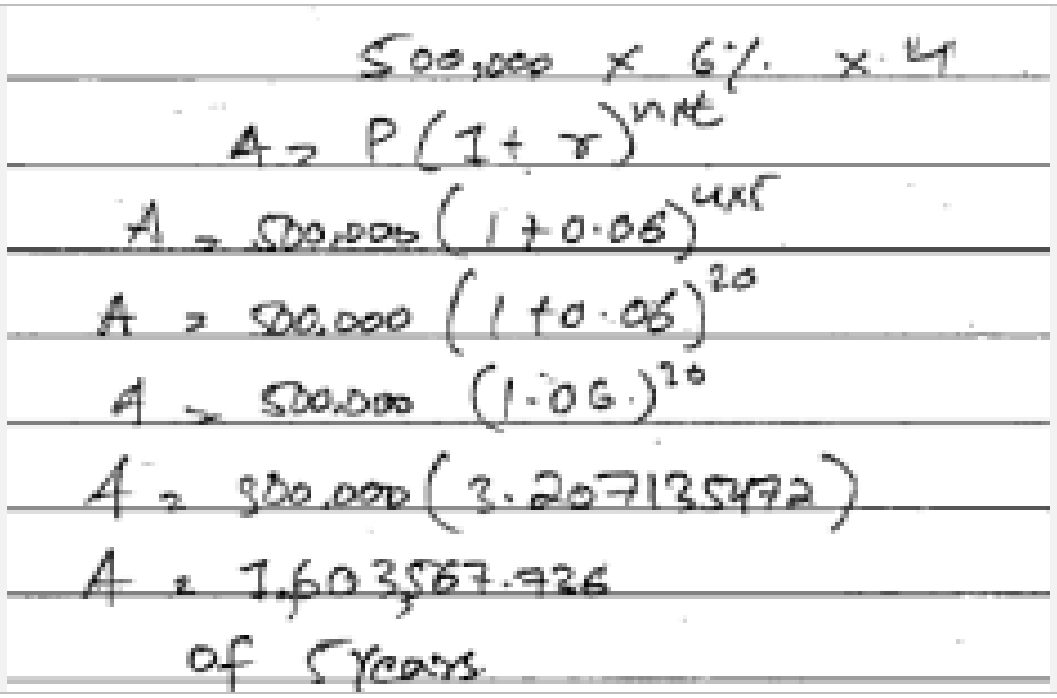
Suggestions for Improvement (Highlighted part)

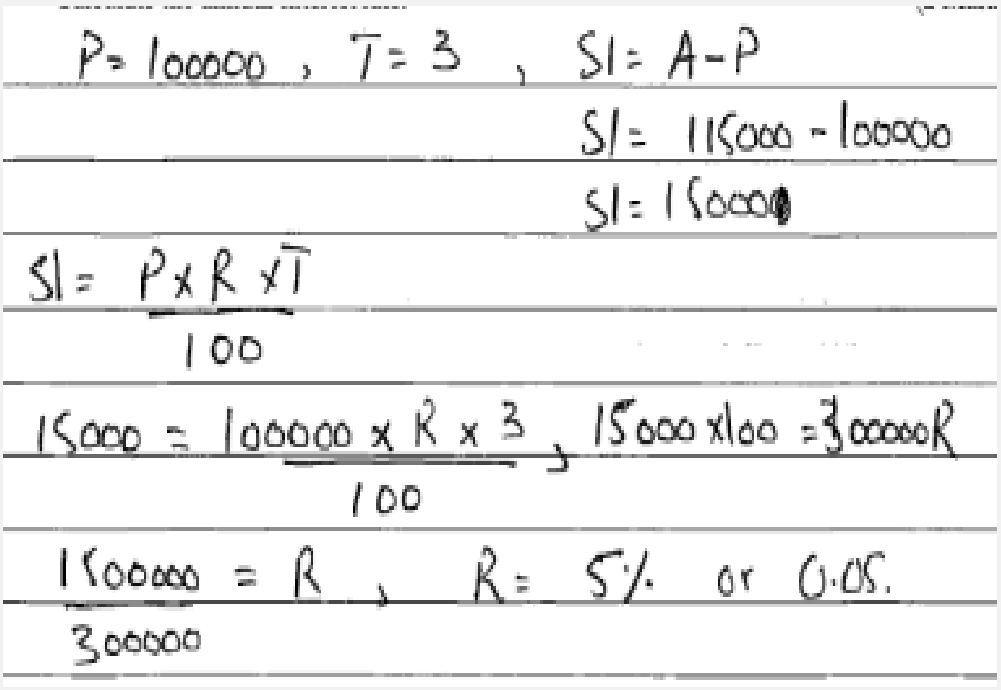
How to Approach SLO	Pedagogy** Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration ** For description of each pedagogy, refer to Annexure A	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

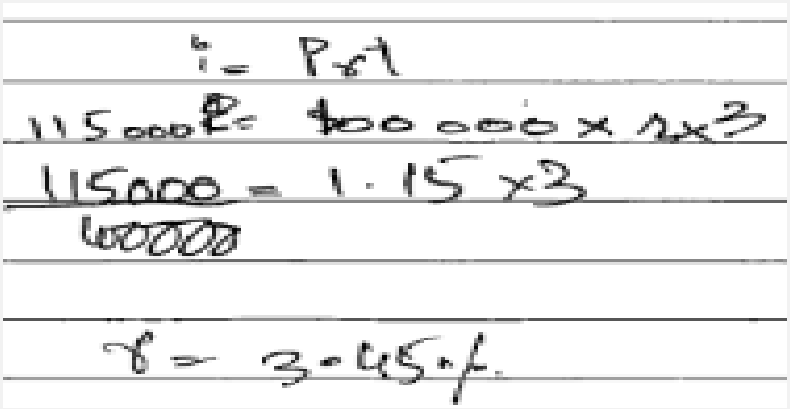
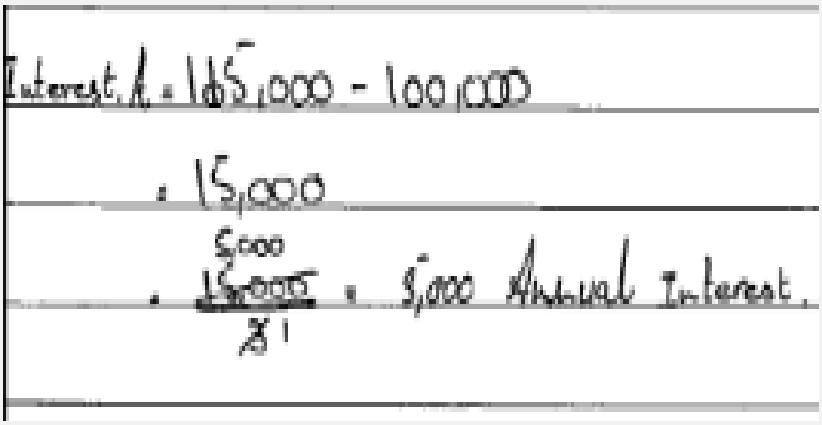
Any Additional Suggestions:
Teachers are advised to use following teaching activities.
Step-by-Step Guidance: Provide step-by-step guidance on how to approach direct proportion problems. Encourage students to clearly define variables and use proportional reasoning.
Comparison Exercises: Create exercises that directly compare direct and indirect proportion scenarios, allowing students to contrast the two concepts and identify their differences.
Real-Life Examples: Provide real-life examples of indirect proportion, such as the relationship between speed and braking distance, where increasing speed leads to longer braking distances.

*K = Knowledge U = Understanding A = Application and other higher-order cognitive skills

Question No. 2ai Candidates were given the choice to attempt any ONE out of the two questions: 2a and 2b.	
Question Text	Saif deposited Rs 500,000 into his bank account. The bank offers compound interest rate of 6% quarterly. Find the amount accumulated in his account after 5 years.
SLO No.	2.2.5
SLO Text	Solve word problems related to the concept of compound interest.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for substituting the correct values in the formula. 1 mark the correct answer.
Overall Performance	In the majority of responses, candidates opted for part 'a'. In such responses, candidates correctly identified the concept of interest (simple and compound) and solved the question accurately by applying the appropriate formula. However, some of the candidates struggled to distinguish between simple and compound interest. Consequently, they faced difficulties in solving the question correctly.
Description of Better Responses	In better responses, candidates demonstrated a strong understanding of the question and its mathematical requirements. They effectively applied the relevant formula, making precise substitutions for both the rate of interest and the required time necessary for accumulating the desired amount. This accurate approach led to the attainment of full marks for their responses

Image of Better Response	
Description of Weaker Responses	In weaker responses, many candidates faced difficulties in understanding the question, especially when it came to applying the appropriate formula. Some mistakenly used the simple interest formula, while others used the correct formula but substituted incorrect values for the interest rate and accumulated time.
Image of weaker Response	

Question No. 2aii	
Question Text	Nisa borrowed Rs 100,000 from her friend Laiba at simple interest rate. She returned Rs 115,000 after 3 years. Calculate the annual interest rate.
SLO No.	2.1.2
SLO Text	Solve word problems related to the concept of simple interest.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for calculating interest. 1 mark for substitution in the formula.
Overall Performance	The candidates approached the question in different ways. Many accurately found the interest and applied the formula correctly, yielding the correct result. However, some candidates used inappropriate methods or formulas, and some failed to subtract the principal and interest amount, resulting in mark deductions.
Description of Better Responses	In the better responses, the majority of candidates successfully solved the question. They first calculated the interest amount accurately and then used this correct value in the formula to find the final result. This showed that they understood the problem correctly. Their approach was careful and followed the rules for calculating interest.
Image of Better Response	
Description of Weaker Responses	In weaker responses, several noticeable issues arose. Firstly, candidates often resorted to inappropriate formulas or methods for performing the required calculations. This indicated a fundamental misunderstanding of the appropriate mathematical approach needed for the problem at hand. Furthermore, even when some candidates correctly applied the relevant formula, they faced challenges in executing the final step accurately. Specifically, they failed to subtract the principal amount from the total, which resulted in errors and marked deductions

Images of Weaker Responses	**Image (i)**  **Image (ii)** 

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestions:
By incorporating these interactive teaching strategies, teachers can help students understand both compound and simple interest, apply the concepts effectively, and recognise their importance in various financial scenarios.
Real-Life Scenarios: Start with real-life examples such as bank accounts, investments, or loans to demonstrate how compound interest works in practical situations.
Simple and Compound Interest Game: Create a game or simulation where students can experiment with different initial investments, interest rates, and compounding frequencies to see how they affect the outcome.
Investment Portfolio Simulation: Set up a classroom investment portfolio simulation where students can choose investments and track their performance over time with compounding.
Compounding Frequency Experiment: Conduct an experiment where students calculate compound interest with different compounding frequencies (e.g., annual, semi-annual, quarterly). Compare the results to illustrate the effects of compounding frequency.
Loan and Savings Scenario: Provide scenarios where students can simulate taking out loans or saving money with simple interest. They can calculate interest payments or earned interest over time.
Interest Rate Discussion: Engage students in discussions about interest rates, including how they are determined, their impact on borrowing and saving, and the role of central banks.
Comparative Analysis: Compare and contrast compound interest and simple interest. Discuss situations where one might be preferable over the other, such as short-term loans or investments.

Question No. 2b Candidates were given the choice to attempt any ONE out of the two questions: 2a and 2b.	
Question Text	Aslam plans to save certain amount of money for his child's higher education in 10 years' time. A famous education fund requires investors to deposit in equal instalments quarterly at interest rate of 6%. If he deposits Rs 5,000 quarterly in this education fund, then calculate the i. number of annuity payments. ii. sum of annuity (accumulated value) after 10 years.
SLO No.	2.3.5
SLO Text	Solve word problems related to the concept of ordinary annuity i.e. investment, deposit and loans.
Max Marks	4
Cognitive Level	A
Checking Hints	i. 1 mark for calculating the number of annuity ii. 1 mark for dividing the interest by 4 to find quarterly interest rate 1 mark for substitution in the formula for accumulated amount 1 mark for evaluation of accumulated amount
Overall Performance	A lesser numbers of candidates opted for this part. In this question, candidates demonstrated a clear understanding of annuities, successfully calculating multiple annuities. Moreover, they adeptly divided the interest and precisely applied the values in the appropriate formula. However, some candidates faced difficulties in calculating the number of annuities and struggled with appropriate interest division.

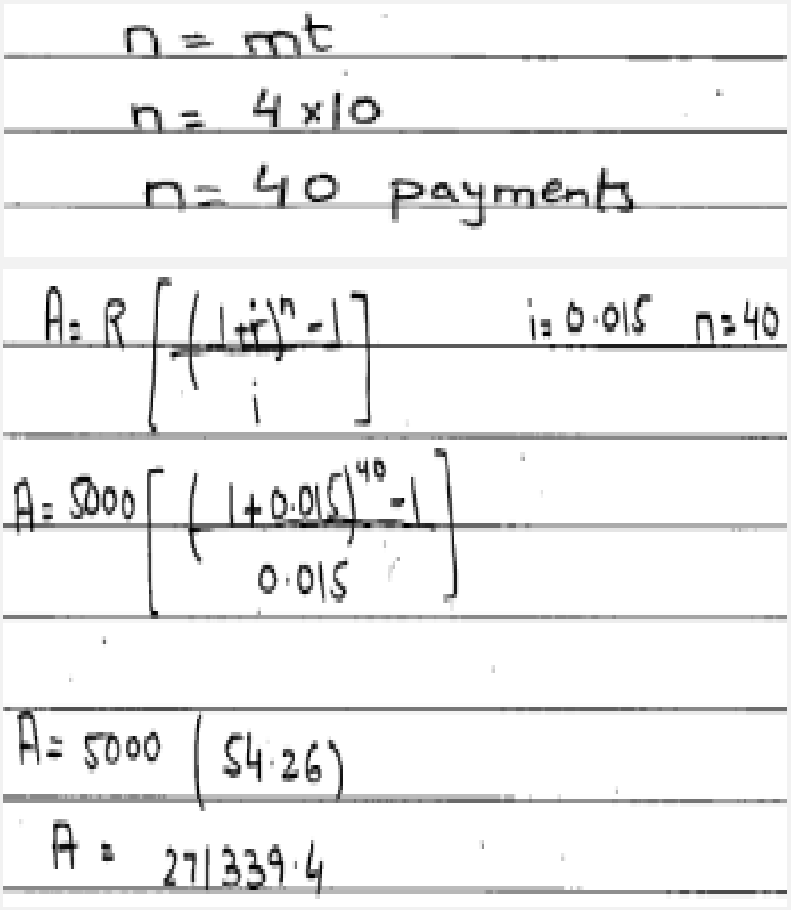
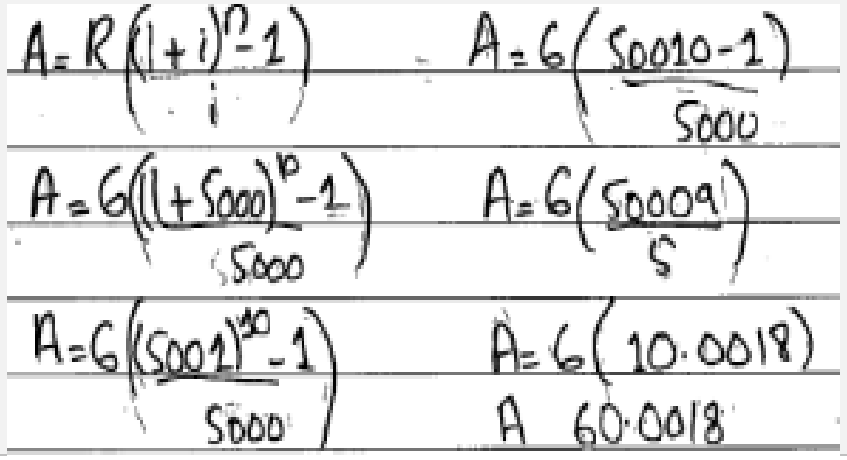
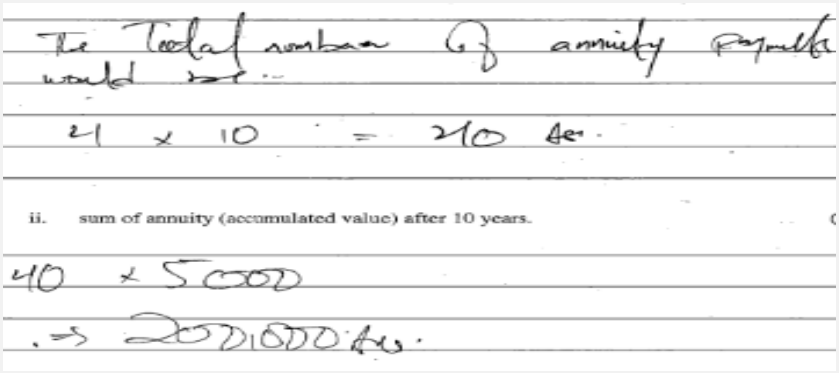
Description of Better Responses	In better responses, candidates demonstrated a clear understanding of annuities and successfully calculated multiple annuities. Moreover, they adeptly divided the interest and precisely utilised the values in the appropriate formula.
Image of Better Response	
Description of Weaker Responses	Weaker responses, demonstrated an inability to accurately determine the number of annuities involved in the problem. Such candidates faced issues when trying to apportion the interest among the annuities. This struggle further emphasised their challenges in applying the appropriate mathematical procedures essential for solving such problems.
Images of Weaker Responses	**Image (i)** 

Image (ii)



Suggestions for Improvement (Highlighted part)

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Any Additional Suggestion:

Following are some interactive teaching strategies for this concept.

Begin by presenting real-life scenarios where ordinary annuities are commonly encountered, such as mortgages, car loans, retirement savings, or insurance policies.

Provide word problems that require students to apply the concepts of ordinary annuities, such as determining monthly mortgage payments, retirement savings contributions, or loan repayments.

Divide students into small groups and assign each group a different type of ordinary annuity problem (e.g., a car loan, a mortgage, a retirement plan). Let them discuss and solve the problem together.

Explore how changes in interest rates impact ordinary annuities. Discuss the implications of higher or lower interest rates on investments, deposits, and loans.

Provide multiple word problems that involve ordinary annuities with varying parameters (interest rates, time periods, payment frequencies). Ask students to compare and contrast these scenarios.

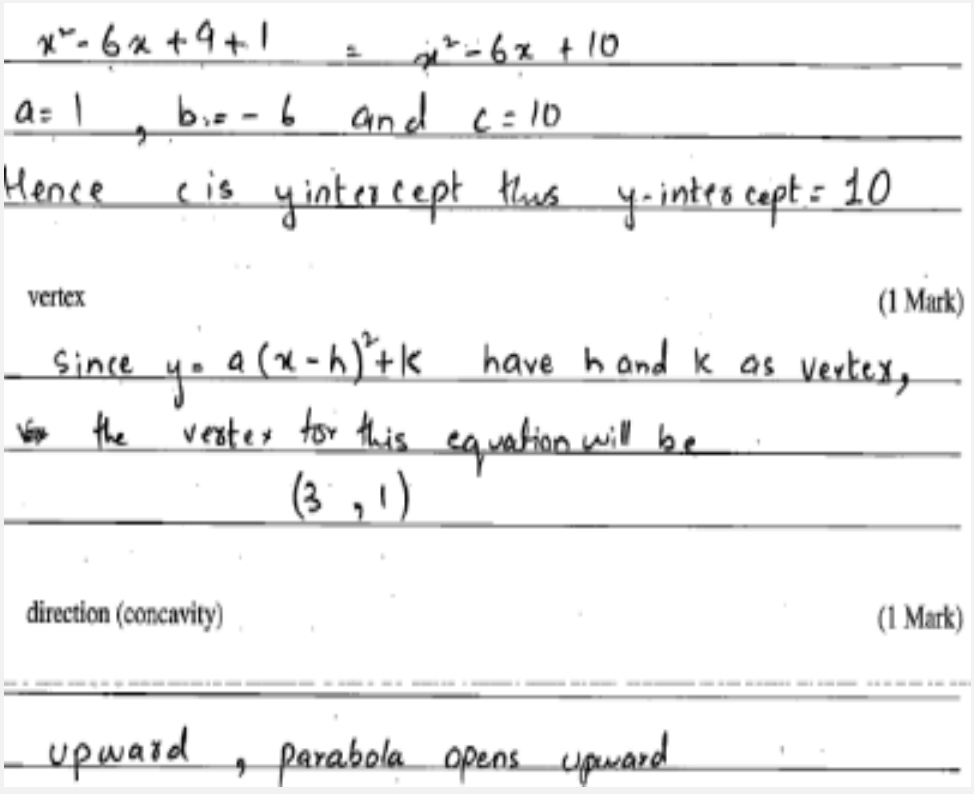
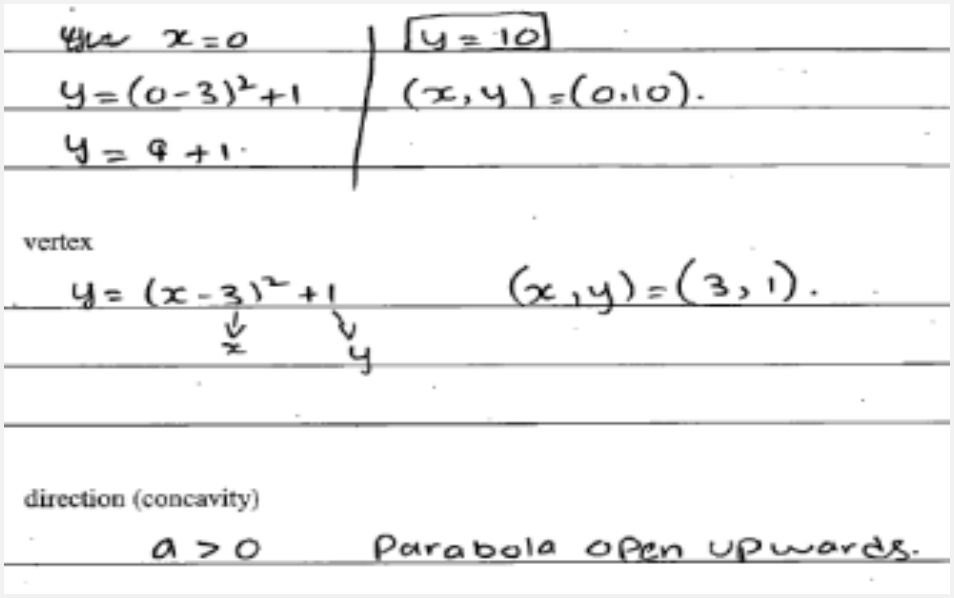
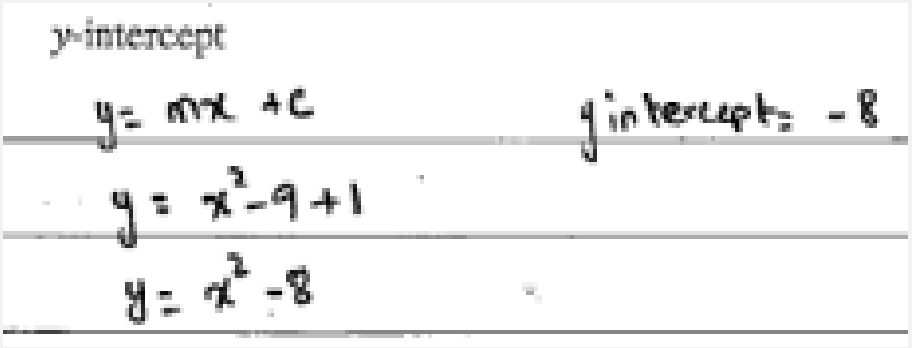
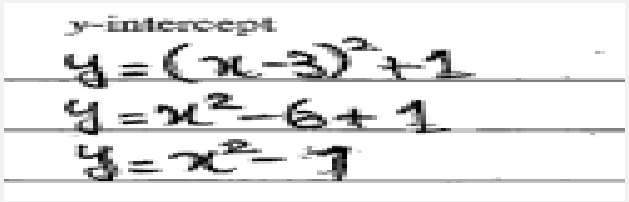
Question No. 3	
Question Text	For the parabola $y = (x-3)^2 + 1$, find the i. *y*-intercept ii. vertex iii. direction (concavity)
SLO No.	4.2.3
SLO Text	Find the vertex, intercepts, axis of symmetry and direction (concavity) of a quadratic function.
Max Marks	3
Cognitive Level	A
Checking Hints	i. 1 mark for correct answer ii. 1 mark for correct answer iii. 1 mark for correct answer
Overall Performance	Most of the candidates showed a clear understanding of finding *Y*-intercept and determining the vertex of a quadratic equation using formulae or comparison method. They also demonstrated knowledge of positive and negative coefficients of X^2. However, other candidates struggled to differentiate between *X*-intercept and *Y*-intercept, failed to find the vertex of the quadratic equation, and lacked understanding of the concept of the coefficient of X^2, leading to difficulties in determining the direction of the parabola.
Description of Better Responses	The candidates displayed a clear understanding of the concept by accurately obtaining the *Y*-intercept, vertex of the parabolic equation, and correctly identifying the direction of the parabola.
Images of Better Responses	**Image (i)**  $x^2 - 6x + 9 + 1 = x^2 - 6x + 10$ $a = 1$, $b = -6$ and $c = 10$ Hence c is y intercept thus y-intercept = 10 vertex (1 Mark) Since $y = a(x-h)^2 + k$ have h and k as vertex, the vertex for this equation will be (3, 1) direction (concavity) (1 Mark) upward, parabola opens upward

	Image (ii) 
Description of Weaker Responses	In weaker responses, candidates exhibited difficulty in identifying the similarities between the parabolic and quadratic equations. Additionally, they faced challenges in accurately finding the quadratic equation's vertex and determining the parabola's direction.
Images of Weaker Responses	**Image (i)**  **Image (ii)**  **Image (iii)**

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
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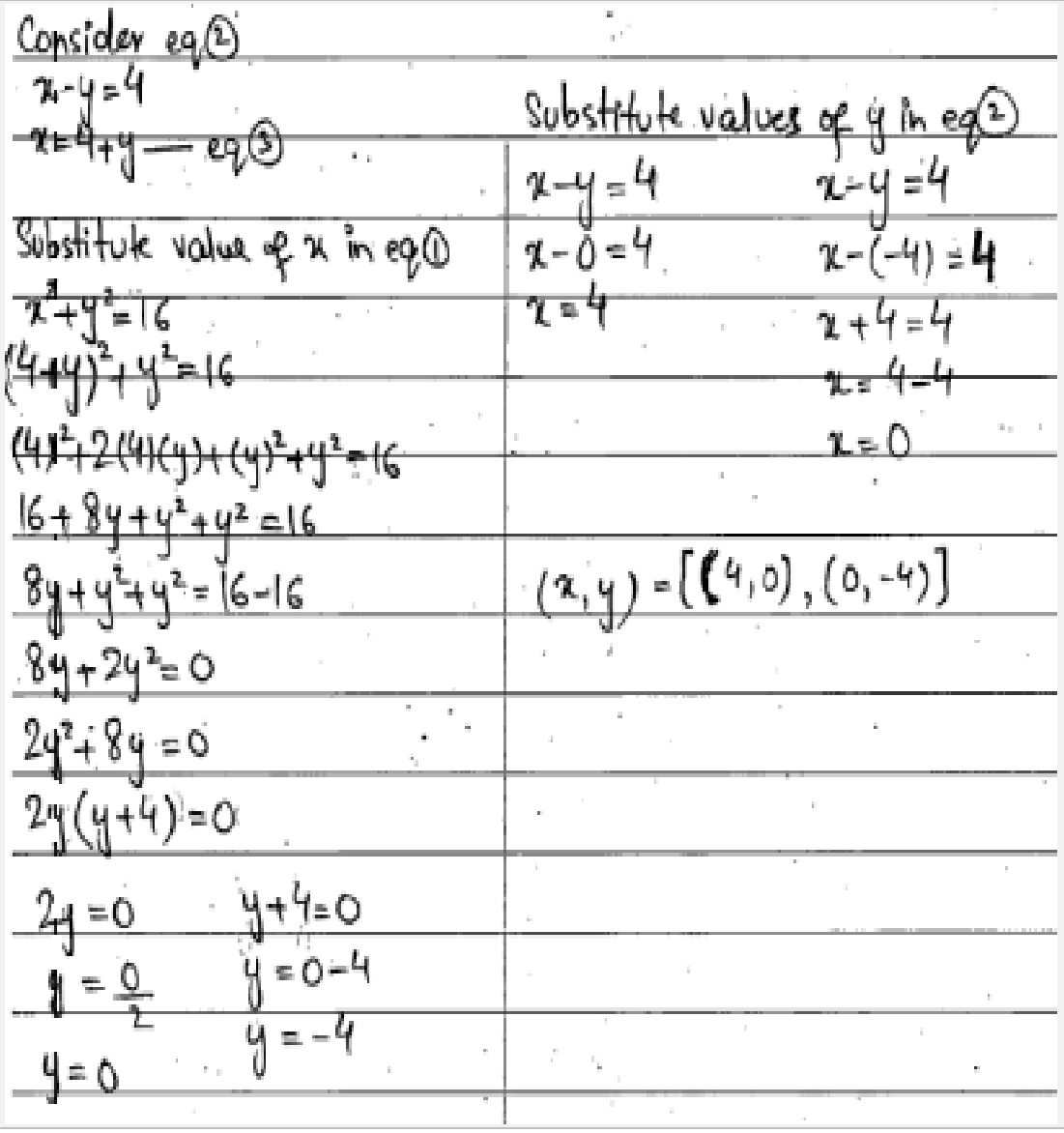
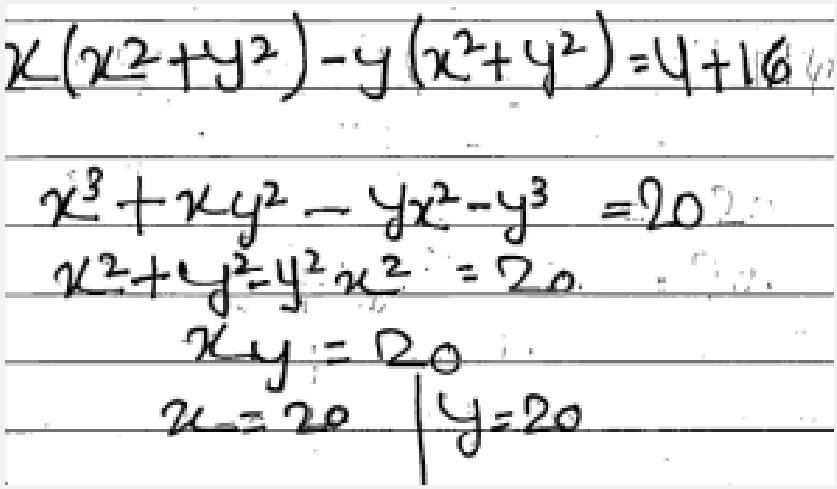
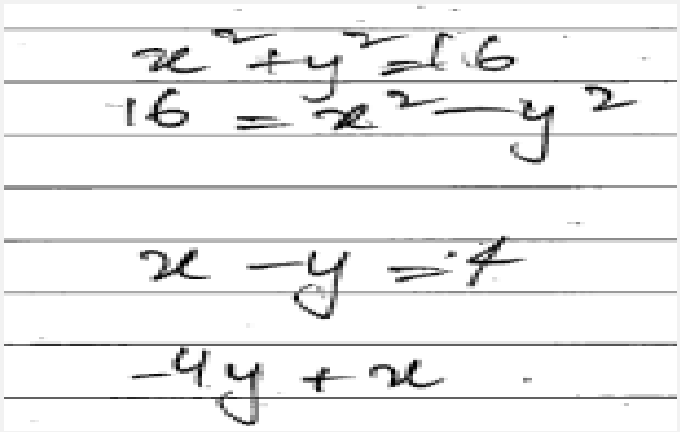
Any Additional Suggestion:
Following are some interactive teaching strategies for this concept.
Start with a visual representation of a quadratic function's graph. Use graphing software or a whiteboard to illustrate the key elements such as vertex, axis of symmetry, and intercepts. Utilise Desmos or similar online graphing platforms that offer pre-made activities and lessons for exploring quadratic functions.
Encourage students to explore various quadratic functions by changing coefficients and observing the effects on the graph. This can help them discover patterns and relationships.
Present real-world scenarios where quadratic functions are used, such as calculating the trajectory of a projectile or modeling profit in a business. Show how understanding these functions is practical

Question No. 4a Candidates were given the choice to attempt any ONE out of the two questions: 4a and 4b	
Question Text	Solve the simultaneous equations given as $x^2 + y^2 = 16$ $x - y = 4$
SLO No.	5.1.1c
SLO Text	Solve simultaneous equations; a. both are linear b. both are quadratic, i.e., $ax^2 + by^2 = m$, $cx^2 + dy^2 = n$ c. one is linear and other is quadratic, i.e., $ax + by = m$ $cx + dy = n$
Max Marks	3
Cognitive Level	A

Checking Hints	1 mark for the substitution for *y* in the quadratic equation using the linear 1 mark for finding the values of *x* 1 mark for finding the corresponding values of *y*
Overall Performance	Majority of the candidates attempted this part, in which many of the candidates accurately substituted the values of *x* and *y* and found their corresponding values However, some candidates faced challenges with substituting the values and failed to find the complete set of points in the solution resulting in deduction of marks.
Description of Better Responses	In better responses, candidates demonstrated a strong command of algebraic concepts and problem-solving skills. They precisely substituted the values of both *x* and *y* from the linear equation into the quadratic equation. Such responses, successfully completed the square and determined the corresponding values of *x* and *y*.
Image of Better Response	
Description of Weaker Responses	In weaker responses, candidates encountered challenges in selecting the appropriate equation to find the values of *x* and *y*. A common issue observed was the tendency of many candidates to attempt to substitute the quadratic equation into the linear equation, resulting in an incorrect approach that did not lead to a solution.

Images of Weaker Responses	**Image (i)**  $x(x^2+y^2)-y(x^2+y^2)=4+16$ $x^3+xy^2-yx^2-y^3=20$ $x^2+y^2-y^2x^2=20$ $xy=20$ $x=20 \mid y=20$ **Image (ii)**  $x^2+y^2=16$ $16=x^2-y^2$ $x-y=4$ $-4y+x$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
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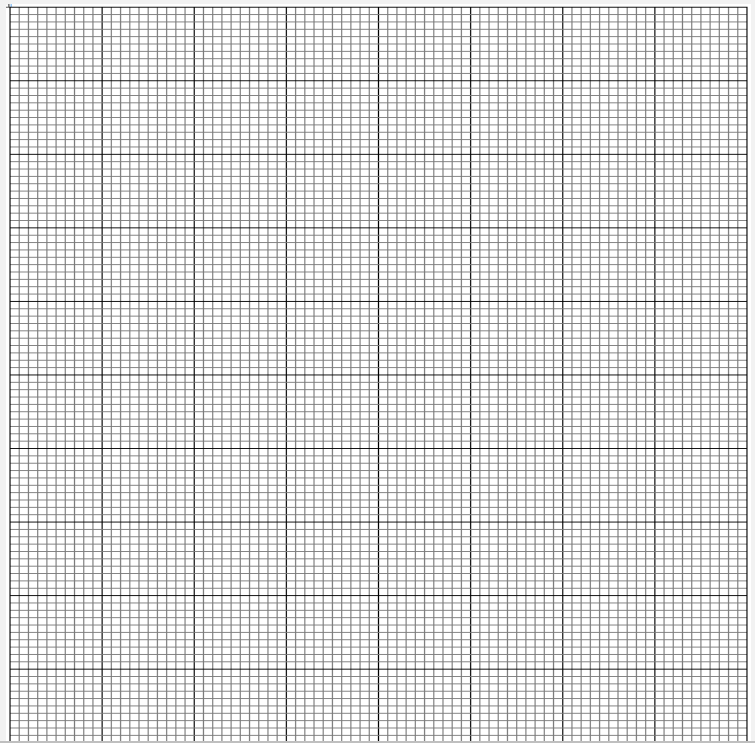
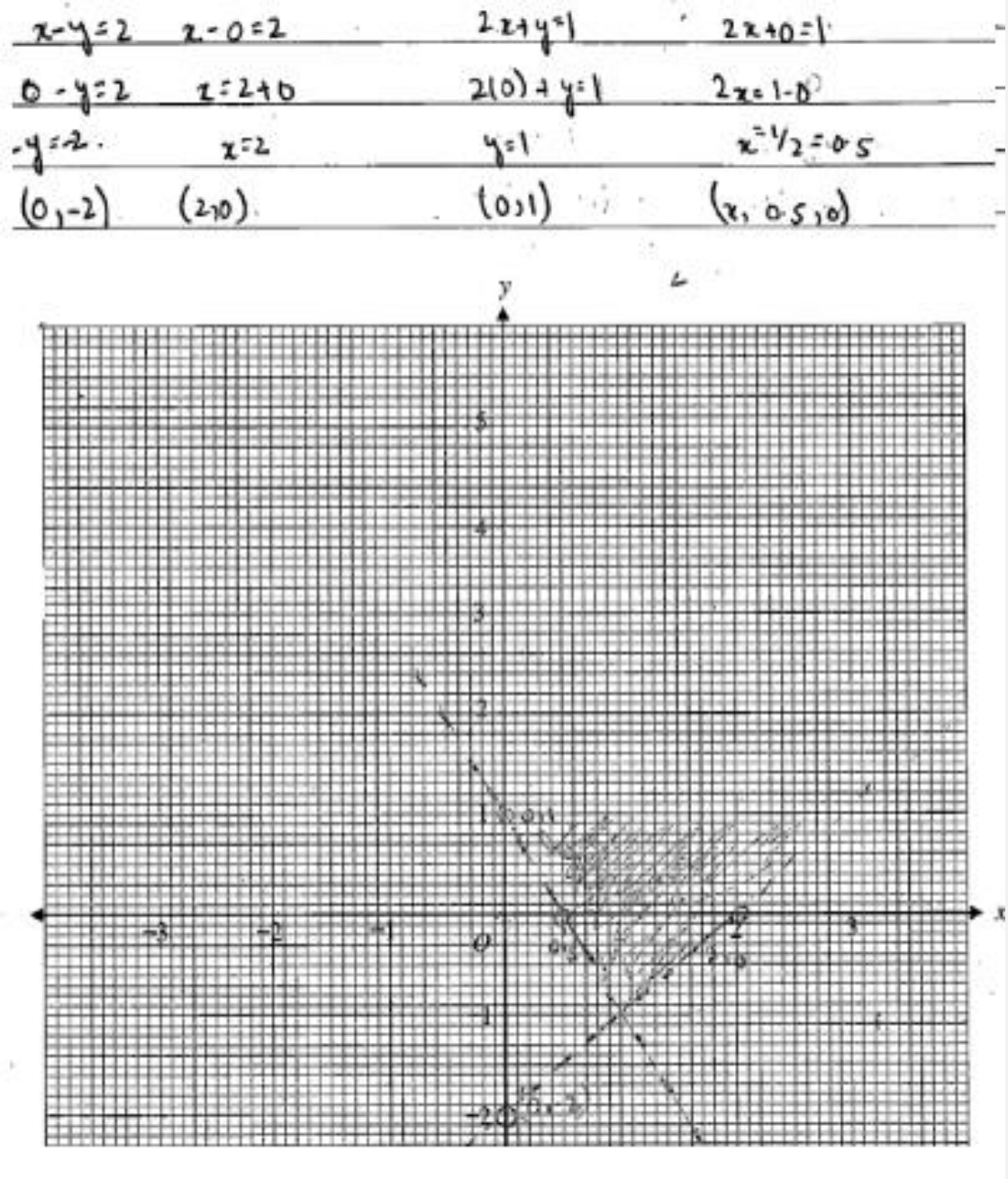
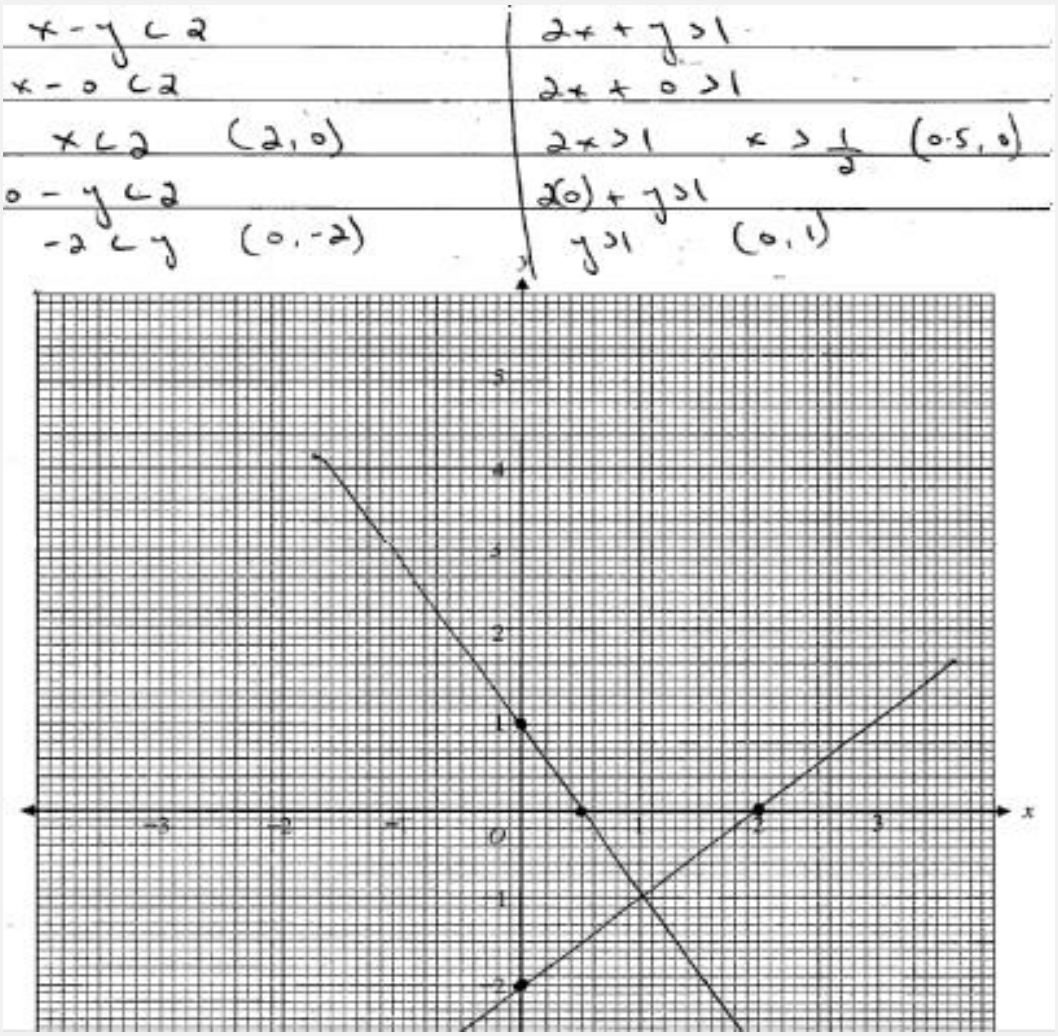
Any Additional Suggestion:
Following are some teaching strategies related to misconceptions and common errors which candidates exhibited.
Step-by-Step Guidance: Provide a detailed, step-by-step guide on how to complete the square, emphasising the purpose of each step.
Interactive Software: Utilise interactive software that allows students to practice completing the square with immediate feedback i.e., Desmos and wolfalpha. Use graphical representations of equations to help students visually identify linear and quadratic equations within the system.
Word Problems: Present word problems that require students to translate real-world situations into equations, fostering a better understanding of equation selection. Provide various examples where students must discern which equations to use based on context, reinforcing the skill.
Error Analysis: Encourage students to analyse their mistakes when substituting values and identify where they went wrong.
Check and Verify: Teach students to check their substitutions by plugging the obtained values back into both equations to ensure consistency.

Question No. 4b

Candidates were given the choice to attempt any ONE out of the two questions: 4a and 4b.

Question Text	In the given grids, show the solution region of $x - y < 2$ and $2x + y > 1$. 
SLO No.	5.2.4
SLO Text	Solve problems based on linear inequalities in two variables.
Max Marks	3

Cognitive Level	A
Checking Hints	1 mark showing the solution region of the first inequality 1 mark for finding the solution region of the second inequality 1 mark for showing the solution region satisfying both
Overall Performance	Fewer candidates correctly solved this part involving inequalities. Most of the candidates effectively differentiated between inequalities, accurately created their corresponding graphs, and correctly identified the feasible region However, some responses struggled with differentiating dotted and permanent lines and faced challenges in drawing them on the graph paper, which led to mistakes in determining the feasible region.
Description of Better Responses	In better responses, candidates accurately understood distinctions between less than, greater than, less than or equal to, and greater than or equal to, and graphed the lines correctly. They found the *XY*-intercepts and determined the feasible region.
Image of Better Response	

Description of Weaker Responses	In weaker responses, candidates struggled to identify the region of the graph (*XY*-intercept) and lacked understanding of different types of inequalities, leading to difficulties in finding the feasible region.
Image of Weaker Response	

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating)	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources		

Any Additional Suggestion:

By implementing these teaching methodologies and strategies, teachers can create an engaging and effective learning environment that promotes a thorough understanding of polynomial equations and complex number factorisation.

Use visual aids and real-life examples to illustrate the meaning and significance of each type of inequality.

Walk students through the process of graphing linear inequalities, including identifying the x and y-intercepts.

Provide guided examples and exercises where students determine the feasible region graphically. Present word problems that require students to translate real-world scenarios into systems of inequalities and find feasible regions.

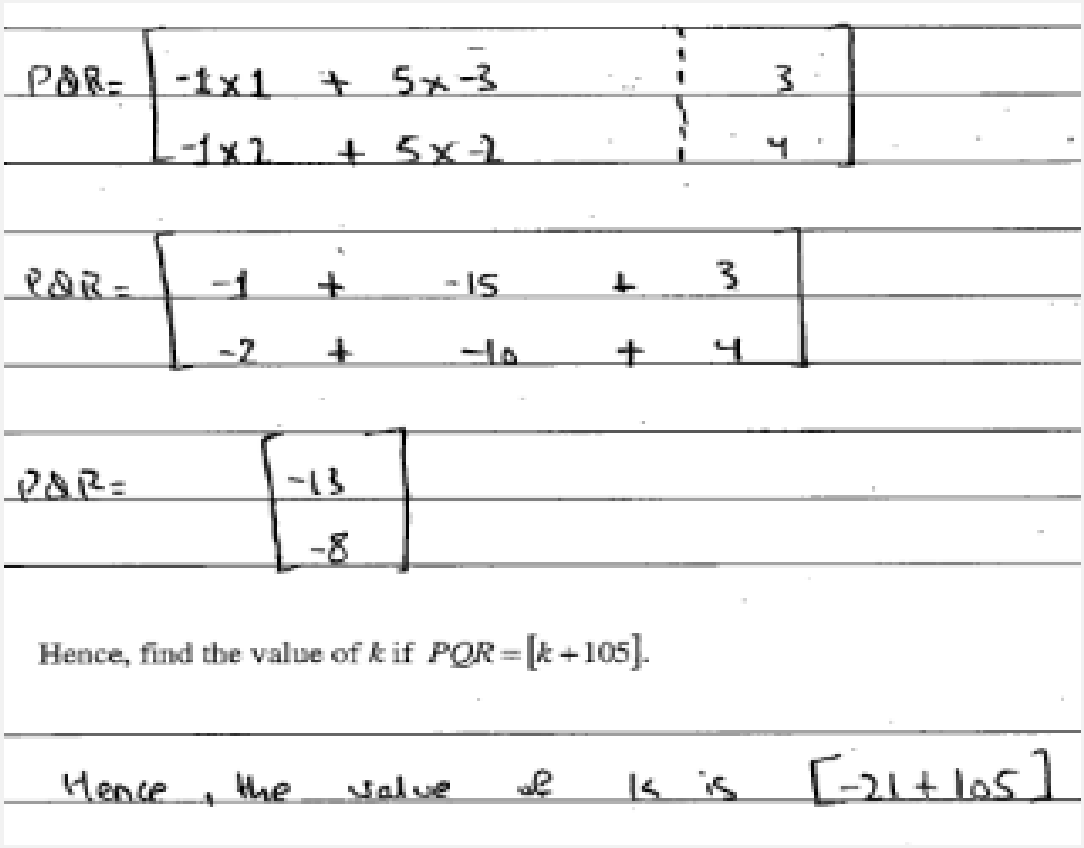
Utilise graphing calculators or online graphing tools to facilitate graphical representation of inequalities, helping students visualise solutions more easily.

Connect the concept of feasible regions and inequalities to practical situations in economics, optimiation, or engineering to highlight their relevance.

Question No. 5	
Question Text	It is given that $P = \begin{bmatrix} -1 & 5 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix}$ and $R = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$. i. Find the matrix PQR ii. Hence, find the value of k if $PQR = \begin{bmatrix} k+105 \end{bmatrix}$.
SLO No.	6.2.2
SLO Text	Find the multiplication of two (or three) matrices.
Max Marks	3
Cognitive Level	A
Checking Hints	i. 1 mark for finding PQ 1 mark for finding PQR ii. 1 mark for writing $PQR = \begin{bmatrix} p+105 \end{bmatrix} = \begin{bmatrix} 55 \end{bmatrix}$
Overall Performance	In question 5, candidates were required to apply the rule of matrix multiplication. In most of the responses, they understood the concept of multiplying matrices, even for non-square matrices, by performing columns to row multiplication. Such responses correctly compared the resulting matrices and found the value of the unknown (p). However, some of the responses showed difficulty in matrix multiplication due to lack of practice. They struggled to determine the correct order of matrices, leading to errors in comparison and finding the value of the unknown.
Description of Better Responses	In better responses, candidates accurately multiplied the row and column matrices and then correctly multiplied the result with a square matrix. This process led to a matrix that easily facilitated comparison.

Image of Better Response	$PQR = \begin{bmatrix} -1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $PQR = \begin{bmatrix} -1+10 & 3+(-10) \end{bmatrix} \times \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $PQR = \begin{bmatrix} 9 & -7 \end{bmatrix} \times \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $PQR = \begin{bmatrix} 27+28 \end{bmatrix}$ $PQR = [55]$ Hence, find the value of k if $PQR = [k+105]$. $PQR = [k+105]$ $[55] = [k+105]$ $55 - 105 = k$ $k = -50$
Description of Weaker Responses	In weaker responses, candidates struggled in correctly performing the row-column matrix multiplication and faced difficulties in matrix comparison.
Images of Weaker Responses	**Image (i)** i. Find the matrix PQR. (2 Marks) $P = [-1 \quad 5] \times Q \begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix} \times R \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ first we find $P \times Q$ then $PQ \times R$: $P = [-1 \quad 5]$ $Q = \begin{bmatrix} 1 & -3 \\ 2 & -2 \end{bmatrix}$ $[(-1 \times 1) + (5 \times 2) \quad (-1 \times -3) + (5 \times -2)] = [1 + 10 \quad -3 + (-10)]$ $PQ [11 \quad -13]$ $PQ [11 \quad -13] \times R \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $[(11 \times 3) + (13 \times (-4))]$ $PQR = [33 \quad -52]$ iii. Hence, find the value of k if $PQR = [k+105]$. (1 Mark) $PQR = [k + 105]$ $[33 \quad -52] = [k + 105]$ $k + 105 = 33$ $k = 33 - 105$

Image (ii)



Suggestions for Improvement (Highlighted part)

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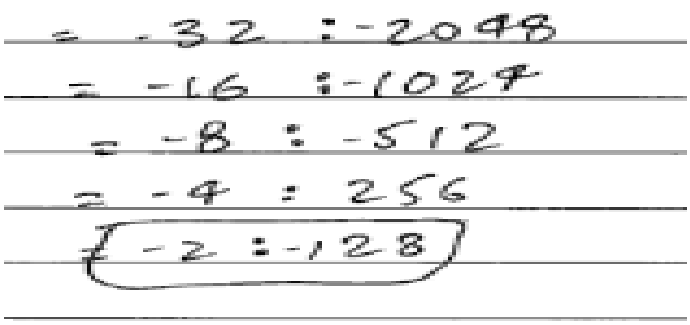
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Knowledge Platform videos • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
To enhance students' understanding of matrix multiplication here are a few teaching methodologies which teachers may use.
Break down the multiplication process into clear steps: For each element of the resulting matrix, calculate the dot product of the corresponding row in the first matrix and the corresponding column in the second matrix. Emphasise the importance of maintaining the order of operations (rows × columns).
Matrix Calculator Tools: Introduce matrix calculator tools or software that allow students to practice matrix multiplication and check their answers. These tools can help students verify manual calculations.
Practice Variety: Provide a variety of matrix multiplication problems, including those involving two or three matrices, to enhance problem-solving skills and adaptability. Include problems with different dimensions to expose students to various scenarios.
Matrix Chain Multiplication: When introducing multiplication of three matrices, explain the concept of matrix chain multiplication. Show how to determine the optimal order of multiplication for a series of matrices to minimize computational complexity.
Error Analysis: Review common errors in matrix multiplication, such as dimension mismatches or incorrect dot products. Encourage students to analyse their mistakes and learn from them.

Question No. 6a Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.	
Question Text	The 5^{th} and 11^{th} terms of a geometric progression are -32 and -2048 respectively. Find the value of the common ratio and the first term of the sequence.
SLO No.	8.3.2
SLO Text	Solve problems involving geometric sequence.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for the substitution in the formula $a_n = ar^{n-1}$ 1 mark for finding the values *of r* 1 mark for finding the value of *a*
Overall Performance	Fewer candidates opted for this part as compared to part 'b'. It involved identifying the geometric series, and in most of the responses, candidates correctly identified the series and found the common ratio and first term by dividing the given series. In contrast, some of the candidates struggled to identify the series and used incorrect series formulas to find the first term and common ratio, some even attempting to use the formulae of general arithmetic terms or sum of arithmetic series.
Description of Better Responses	In better responses, candidates accurately identified the series, correctly divided the indicated terms, and applied the geometric series formula to find the first term and the common ratio.

Image of Better Response	$a_1r^4 = -32$ — ① $a_1r^{10} = -2048$ — ② Dividing eq ② by ① $\frac{ar^{10}}{ar^4} = \frac{-2048}{-32}$ $r^6 = 64$ when $2^6 = 64$ thus common ratio = 2 For a_1 using eq ① $a_1(2)^4 = -32$ $a_1(16) = -32$ $a_1 16 = -32$ $a_1 = \frac{-32}{16}$ $a_1 = -2$
Description of Weaker Responses	In weaker responses, candidates struggled to correctly identify the series, and some even attempted to use formulas as above, which led to incorrect solutions.
Images of Weaker Responses	**Image (i)** $a_5 = a_1 + 4d$ $= 1 + 4(-0.9)$ $= -2.6$ $a_8 = a_1 + 7d$ $= 1 + 7(-0.9)$ $= -5.3$ $a_6 = a_1 + 5d$ $= 1 + 5(-0.9)$ $= -3.5$ $a_9 = a_1 + 8d$ $= 1 + 8(-0.9)$ $= -6.2$ $a_7 = a_1 + 6d$ $= 1 + 6(-0.9)$ $= -4.4$ a) $a_5 = a_1 + 4d$ $-32 = a_1 + 4d$ ...(i) $a_1 = -4d + 32$ $a_{11} = a_1 + 10d$ $-2048 = a_1 + 10d$ subtract $a_1 = -10d + 2048$ $a_1 = -4d + 32$ $6d = 2016$ $d = \frac{2016}{6}$ $d = 336$ $a_5 = a_1 + 4(336) = a_1 = -1342$

	Image (ii) 

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Any Additional Suggestion:

Following are some interactive teaching strategies for teaching geometric sequences that teachers may use.

Technology Tools: Utilise technology tools like spreadsheet software (e.g., Excel or Google Sheets) to generate and visualise geometric sequences. Show how changing initial values and ratios affects the sequence.

Geometric Sequence Explorer: Use online interactive tools or apps i.e., omnicalculator and Symbolab that allow students to explore geometric sequences by inputting initial values and ratios. These tools can provide immediate feedback.

Comparing Sequences: Provide multiple geometric sequences with different initial values and ratios. Have students compare and contrast these sequences, discussing their growth rates.

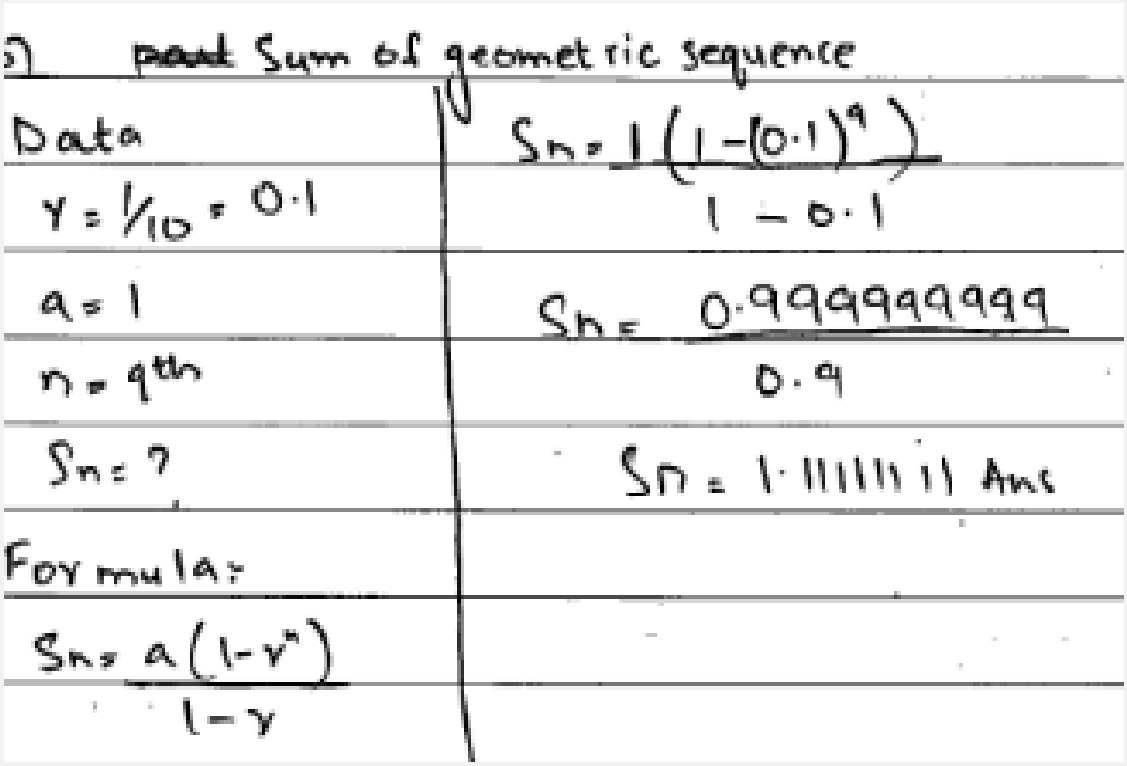
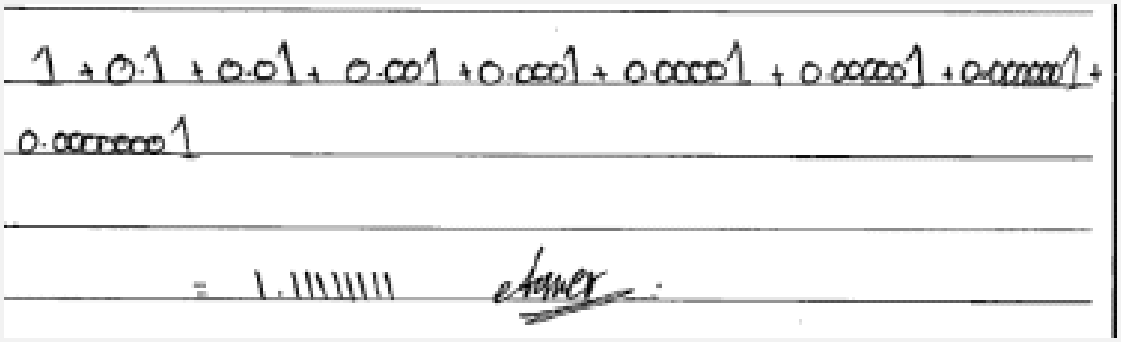
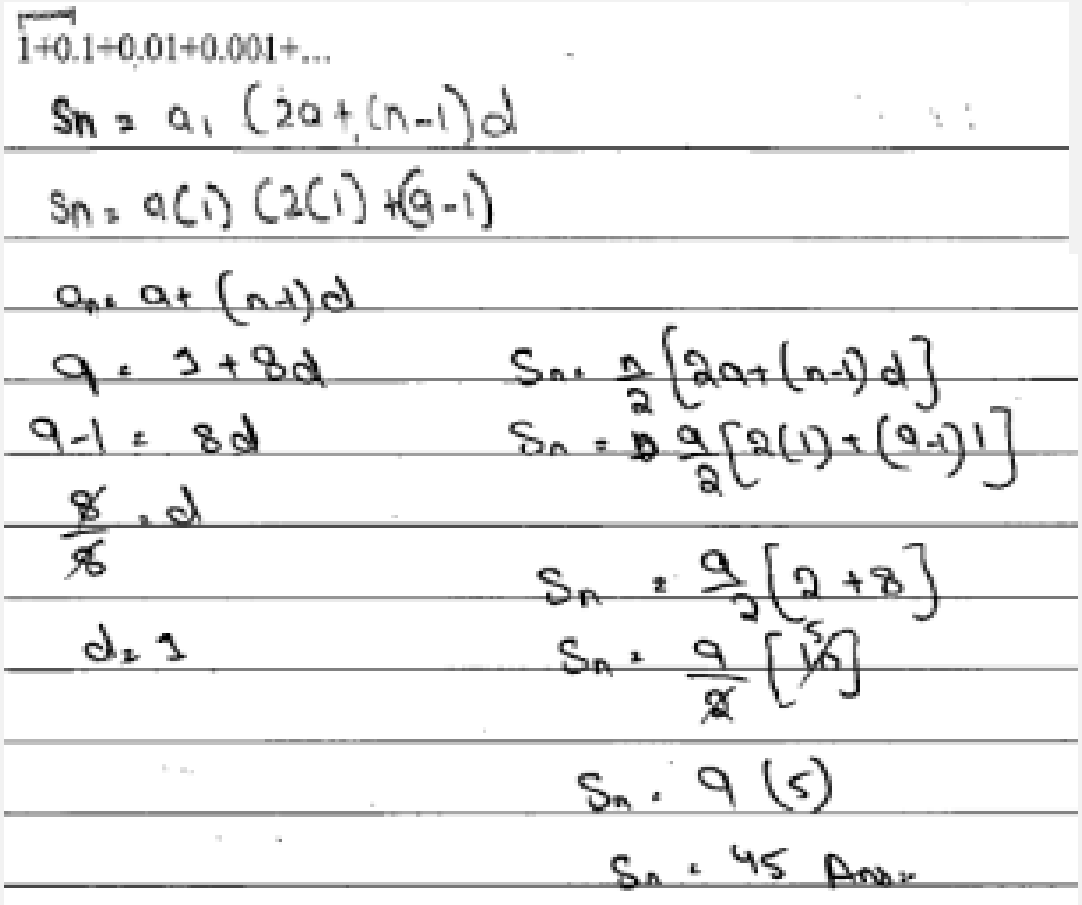
Question No. 6b Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.	
Question Text	Find the sum of the given geometric series up to the 9th term. 1+0.1+0.01+0.001+…
SLO No.	8.4.2
SLO Text	Find the sum to *n* terms of a finite geometric series.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for identification of *a*, *r* and *n* 1 mark for the application of the formula $S_n = \frac{a(1-r^n)}{1-r}$ 1 mark for finding the required sum
Overall Performance	Most candidates opted for this part. The question was related to sum of geometric series. Candidates correctly identified the series, applied the appropriate formula for sum of series based on the common ratio, and found the sum of the indicated terms accurately. In addition, some of the candidates used the direct t approach by manually calculating the series up to the 9th term and summing the values for the correct result. In contrast, some responses showed difficulties in identifying the geometric series. While some candidates mistakenly used irrelevant formula of sum of arithmetic series, leading to incorrect results.
Description of Better Responses	In better responses, the candidates correctly identified the geometric series, were able to select the correct formula for solution in the basis of the value of r (less or greater than 1) and found the indicated term. In some cases, candidates directly extended the given series, reached to the 9th term and summed up the series to find the indicated result.
Images of Better Responses	**Image (i)**  5) Sum of geometric sequence Data $r = 1/10 = 0.1$ $a = 1$ $n = 9^{th}$ $S_n = ?$ Formula: $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{1(1-(0.1)^9)}{1-0.1}$ $S_n = \frac{0.999999999}{0.9}$ $S_n = 1.111111111$ Ans

	Image (ii) 
Description of Weaker Responses	In weaker responses, candidates struggled to differentiate between arithmetic and geometric series, mistakenly using the formula of sum of arithmetic series. Additionally, some candidates were unaware of the difference between the common ratio (less than or greater than) and applied the formula incorrectly, resulting in inaccurate solution.
Image of Weaker Response	

Suggestions for Improvement (Highlighted part)

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Any Additional Suggestion:

Following are some interactive teaching strategies for teaching sum of geometric series that teachers may use.

Real-World Applications: Relate geometric series to practical scenarios, such as calculating compound interest, amortization schedules, or population growth. Show how geometric series are used in finance, science, and other fields.

Technology Tools: Utilise technology tools like spreadsheet software (e.g., Excel or Google Sheets) to generate and visualise geometric sequences. Show how changing initial values and ratios affects the sequence.

Geometric Sequence Explorer: Use online interactive tools or apps i.e., omnicalculator and Symbolab that allow students to input the initial term, common ratio, and number of terms to calculate the sum of the series. These tools can help students experiment with different series.

Critical Thinking Exercises: Present students with challenging geometric series problems that require creative thinking and problem-solving skills. Encourage them to explore different methods of finding the sum.

Annexure A: Pedagogies Used for Teaching the SLOs

Pedagogy: Storyboard
Description: A visual pedagogy that uses a series of illustrated panels to present a narrative, encouraging creativity and critical thinking. It helps learners organise ideas, sequence events, and comprehend complex concepts through storytelling.
Example: In a Literature class, students are tasked with creating storyboards to visually retell a novel. They draw key scenes, write captions, and present their stories to the class, enhancing their reading comprehension and fostering their imagination.

Pedagogy: Cause and Effect
Description: This pedagogy explores the relationships between actions and consequences. By analysing cause-and-effect relationships, learners develop a deeper understanding of how events are interconnected and how one action can lead to various outcomes.
Example: In a History class, students study the causes and effects of the Industrial Revolution. They research and discuss how technological advancements in manufacturing led to significant societal changes, such as urbanisation and labour reform movements.

Pedagogy: Fish and Bone
Description: A method that breaks down complex topics into main ideas (the fish) and supporting details (the bones). This visual approach enhances comprehension by highlighting essential concepts and their relevant explanations.
Example: During a Biology class on human anatomy, the teacher uses the fish and bone technique to teach about the human skeletal system. Teacher presents the main components of the human skeleton (fish) and elaborates on each bone's structure and function (bones).

Pedagogy: Concept Mapping
Description: An effective way to visually represent relationships between ideas. Learners create diagrams connecting key concepts, aiding in understanding the overall structure of a subject and fostering retention.
Example: In a Psychology assignment, students use concept mapping to explore the various theories of personality. They interlink different theories, such as Freud's psychoanalysis, Jung's analytical psychology, and Bandura's social-cognitive theory, to see how they relate to each other.

Pedagogy: Audio Visual Resources
Description: Incorporating multimedia elements like videos, images, and audio into lessons. This approach caters to different learning styles, making educational content more engaging and memorable.
Example: In a General Science class, the teacher uses a documentary-style video to teach about the solar system. The video includes stunning visual animations of the planets, interviews with astronomers, and background music, enhancing students' interest and understanding of space.

Pedagogy: Think, Pair, and Share
Description: A collaborative learning technique where students ponder a question or problem individually, then discuss their thoughts in pairs or small groups before sharing with the entire class. It fosters active participation, communication skills, and diverse perspectives.
Example: In a Literature in English class, the teacher poses a thought-provoking question about a novel's moral dilemma. Students first reflect individually, then pair up to exchange

their opinions, and finally participate in a lively class discussion to explore different viewpoints.

Pedagogy: Questioning Technique (Socratic Approach)
Description: Based on Socratic dialogue, this method stimulates critical thinking by posing thought-provoking questions. It encourages learners to explore ideas, justify their reasoning, and discover knowledge through a process of inquiry.
Example: In an Ethics class, the instructor uses the Socratic approach to lead a discussion on the meaning of justice. By asking a series of probing questions, the students engage in a deeper exploration of ethical principles and societal values.

Pedagogy: Practical Demonstration
Description: A hands-on approach where learners observe real-life applications of theories or skills. Practical demonstrations enhance comprehension, skill acquisition, and problem-solving abilities by bridging theoretical concepts with real-world scenarios.
Example: In a Food and Nutrition class, the instructor demonstrates the proper technique for filleting a fish. Students observe and then practice the skill themselves, learning the practical application of knife skills and culinary precision.

(**Note:** The examples provided in this annexure serve as illustrations of various pedagogies. It is important to understand that these pedagogies are versatile and can be applied across subjects in numerous ways. Feel free to adapt and explore these techniques creatively to enhance learning outcomes in your specific context.)

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Additionally, we express our gratitude to the esteemed team of reviewers for their constructive feedback on overall performance, better and weaker responses, and validating teaching pedagogies along with suggestions for improvement.

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- Zain Muluk, Manager, Examination Development, AKU-EB
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