AGA KHAN UNIVERSITY EXAMINATION BOARD

Notes from E-Marking Centre HSSC-I Mathematics Annual Examinations 2023

Introduction

This document has been prepared for the teachers and candidates of Higher Secondary School Certificate (HSSC) Part I (Class XI) Mathematics. It contains comments on candidates' responses to the 2023 HSSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that requires candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations

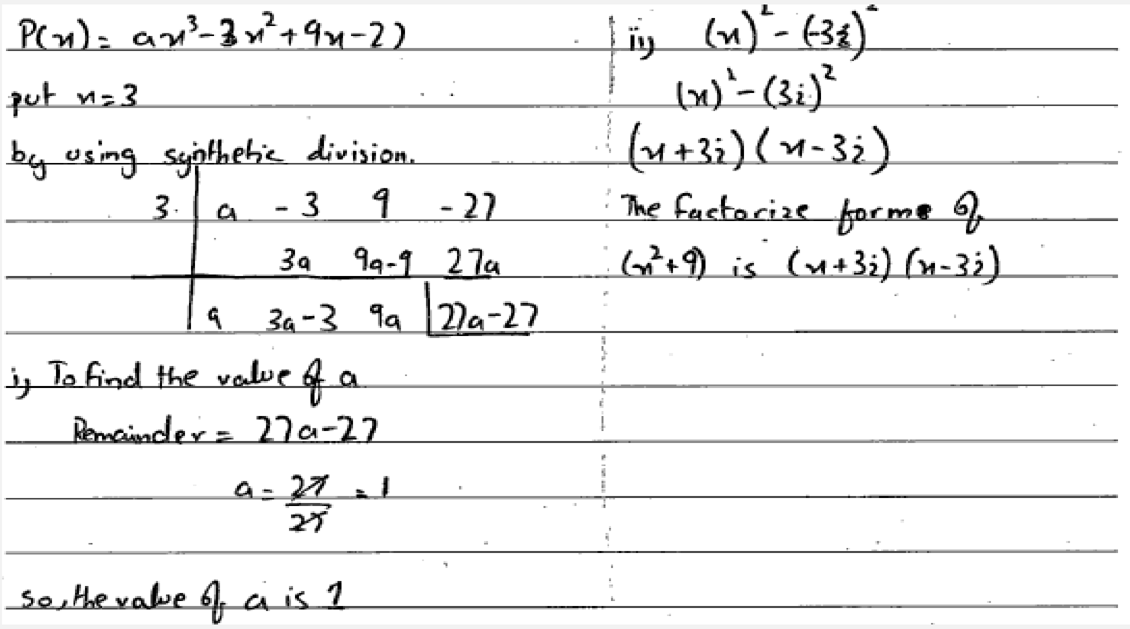
Candidates performed really well in some concepts, such as, Complex Numbers, Matrices and Application of Trigonometric Identities. However, candidates who did not score well mostly failed to understand the demands of the questions, often misinterpreting the command words and the stimuli.

Mentioned below are a few concepts that teachers need to focus so that the candidates may perform better.

- Permutation, Combination and Probabilities
- Roots of the Quadratic Equation
- Concept of Circum-Radius, In-Radius and Escribed Radius
- Table of values of Function for Graph and Plotting

Note: Candidates' responses shown in this report have not been corrected for grammar, spelling, format, or information.

DETAILED COMMENTS
Constructed Response Questions (CRQs)

Question No. 1	
Question Text	For the polynomial $ax^3 - 3x^2 + 9x - 27$, the factors are $(x-3)$ and (x^2+9). i. Find the value of *a*. ii. Hence, factorise (x^2+9).
SLO No.	1.3.2
SLO Text	Factorise the polynomial *P*(*x*), for example. a. $x^2 + y^2 = (x+iy)(x-iy)$ b. $x^3 - 3x^2 + x + 5 = (x+1)(x-2-i)(x-2+i)$
Max Marks	4
Cognitive Level	A*
Checking Hints	i. 1 mark for using appropriate method for finding the value of *a*. 1 mark for stating the value of *a*. ii. 1 mark for writing x^2+9 as $(x)^2 - (3i)^2$. 1 mark for the correct factorisation.
Overall Performance	This question had two parts, with part (i), involving finding the value of an unknown constant *a* in a polynomial and part (ii), focusing on factorisation of complex numbers. While many candidates correctly approached these tasks, some made errors in algebraic manipulations when solving for *a* in part (i), and in part (ii), errors included misapplication of trigonometric identities and overlooking factors during complex number factorisation.
Description of Better Responses	In part (i), candidates applied the synthetic division or factor theorem to find the value of '*a*'. Such candidates successfully evaluated the value of '*a*' by using synthetic division. While in part (ii), candidates converted 9 into complex numbers and applied $a^2 - b^2$ for further factorisation.
Images of Better Responses	**Image (i)** 

$P(x) = ax^3 - 3x^2 + 9x - 27$
put $x = 3$
by using synthetic division.

$$\begin{array}{r|rrrr} 3 & a & -3 & 9 & -27 \\ & & 3a & 9a-9 & 27a \\ \hline & a & 3a-3 & 9a & 27a-27 \end{array}$$

i) To find the value of a
Remainder $= 27a - 27$
$a = \frac{27}{27} = 1$
so, the value of a is 1

ii) $(x)^2 - (-3i)^2$
$(x)^2 - (3i)^2$
$(x+3i)(x-3i)$
The factorize form of (x^2+9) is $(x+3i)(x-3i)$

	Image (ii) (i) applying factor theorem: as $(x-3)$ factor of polynomial $\therefore\ a(3)^3 - 3(3)^2 + 9(3) - 27 = 0$ $27a - 27 + \cancel{27} - \cancel{27} = 0$ $\cancel{27}a = \cancel{27}$ $\therefore\ \boxed{a = 1}$ (ii) $(x^2 + 9)$ $= (x)^2 - (3i)^2$ $= (x + 3i)\ (x - 3i)$
Description of Weaker Responses	Weaker responses showed that candidates met the requirement of the question to a certain extent only. Though majority of the candidates were able to solve part (i) by factor theorem or synthetic division. However, in part (ii), the candidates added 6*x* in the given equation and applied middle term break.
Image of Weaker Response	$ax^3 - 3x^2 + 9x - 27 = 0$ — eq ① $\quad x - 3 = 0$ Put the value of x in eq ① $\quad x = 3$ — eq ② $a(3)^3 - 3(3)^2 + 9(3) - 27 = 0$ $27a - 27 + \cancel{27} - \cancel{27} = 0$ $27a - 27 = 0$ $a = 27/27 = 1$ Factorize (open by $(a+b)^2$) $(x^2 + 6x + 3^2 = 0 \quad x + 3 = 0 \quad 3 \mid 9$ $x^2 + 6x + 9 = 0 \quad x = -3 \quad 3 \mid 3$ $x^2 + 3x + 3x + 9 = 0$ $\quad$ factor $\quad \mid 1$ $x(x+3)\ 3(x+3) = 0 \quad (x+3)$

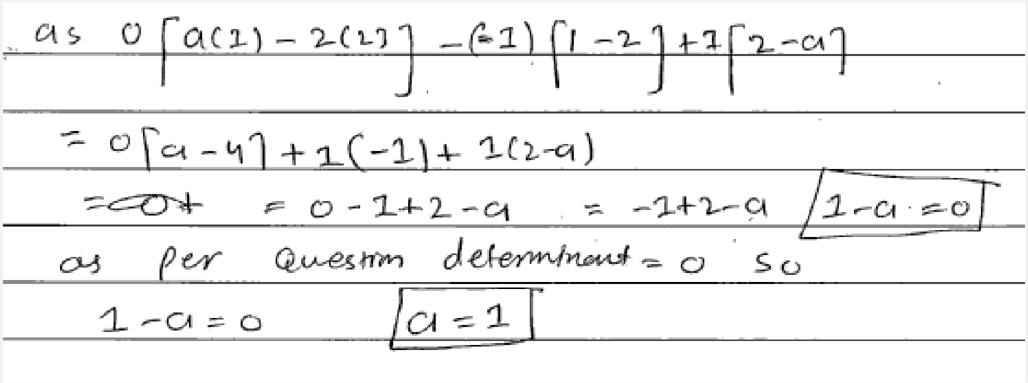
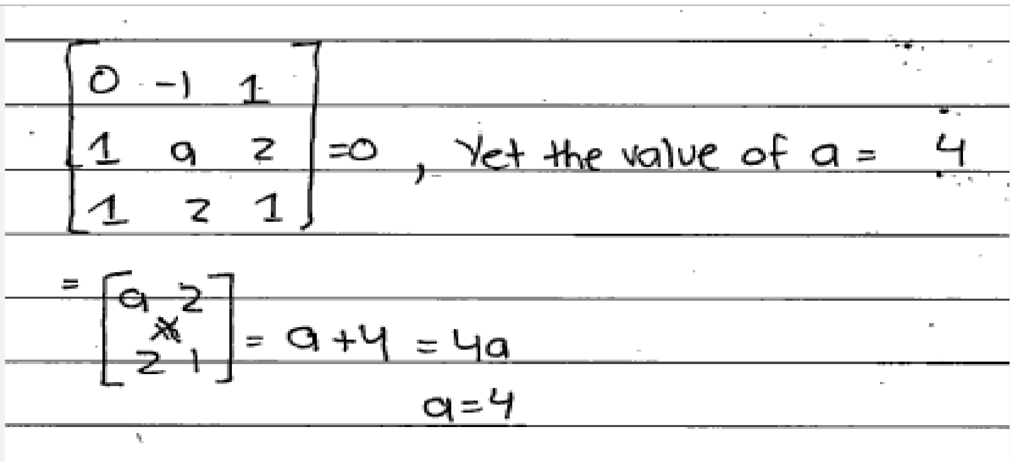
Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy** Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept Refer to the resource guide for extra resources	• Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration ** For description of each pedagogy, refer to Annexure A	
Any Additional Suggestion: By implementing these teaching methodologies and strategies, teachers can create an engaging and effective learning environment that promotes a thorough understanding of polynomial equations and complex number factorisation. Technology Integration: Utilise educational technology and software tools to enhance learning. There are various mathematical software packages that can help visualise and solve polynomial equations and complex number problems. Practice Variety: Offer a wide variety of practice problems, ranging from basic to advanced difficulty levels. Encourage students to practice regularly and gradually increase the complexity		

*K = Knowledge U = Understanding A = Application and other higher-order cognitive skills

Question No. 2i	
Question Text	If the determinant of the matrix $\begin{bmatrix} 0 & -1 & 1 \\ 1 & a & 2 \\ 1 & 2 & 1 \end{bmatrix}$ is 0, then find the value of a.
SLO No.	2.3.4
SLO Text	Solve problems related to singular and non-singular matrices.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for taking determinant 0. 1 mark for expanding the determinant. 1 mark for calculating a.
Overall Performance	It was noted that the question proved to be relatively easy for the vast majority of the candidates. Such candidates who successfully followed the steps to find the determinant and solve for a would have achieved full marks on this question
Description of Better Responses	In better responses, candidates expanded the determinant using *row* 1 and set it equal to zero. By appropriately simplifying the algebraic steps, they determined the value of a.

Image of Better Responses	
Description of Weaker Responses	In weaker responses, candidates confused matrix and determinant concepts. Furthermore, they focused only on a single element from either *row* 1 or *column* 1 to calculate the value of *a*.
Image of Weaker Responses	

Question No. 2ii	
Question Text	A matrix is defined as $M=\begin{bmatrix}0 & 2 & -1\\ 1 & 1 & 2\\ 0 & 1 & 0\end{bmatrix}$. Show that $\lvert 2M\rvert = -8$.
SLO No.	2.4.2
SLO Text	Evaluate the determinant without expansion (using properties of determinants).
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for finding 2*M*. 1 mark for expansion of determinant. 1 mark for evaluating determinant of 2*M* and show equal to -8.
Overall Performance	It was generally observed that the question was regarded as relatively easy by most candidates. Most of the candidates which suggests that they were proficient in performing these determinant calculations and applying determinant properties. Overall, candidates appeared to have a solid grasp of the concepts tested in this question.
Description of Better Responses	In better responses, candidates proved the equation in two ways. Firstly, some candidates multiplied the given matrix by 2 and then calculated the determinant by *row*/ *column* expansion. Secondly, some candidates calculated the determinant of the given matrix and then multiplied with the cube of 2.

Image of Better Response	$2M = \begin{vmatrix} 0 & 4 & -2 \\ 2 & 2 & 4 \\ 0 & 2 & 0 \end{vmatrix}$ Expand by C1 $0 - 2(0+4) - 0$ $0 - 2(4) - 0$ $= -8$ proved
Description of Weaker Responses	In weaker responses, most of the candidates mainly computed determinants without multiplying by 2. Hence, they were not able to prove the given equation.
Image of Weaker Response	$\lvert M\rvert = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ $\quad \frac{adj(M)}{\lvert A\rvert}$ $\lvert M\rvert = 0(0+2) - 2(0-0) - 1(1-0)$ $\lvert M\rvert = 0 \quad - 0 \quad - 1 \quad 0$ $\lvert M\rvert = -1$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

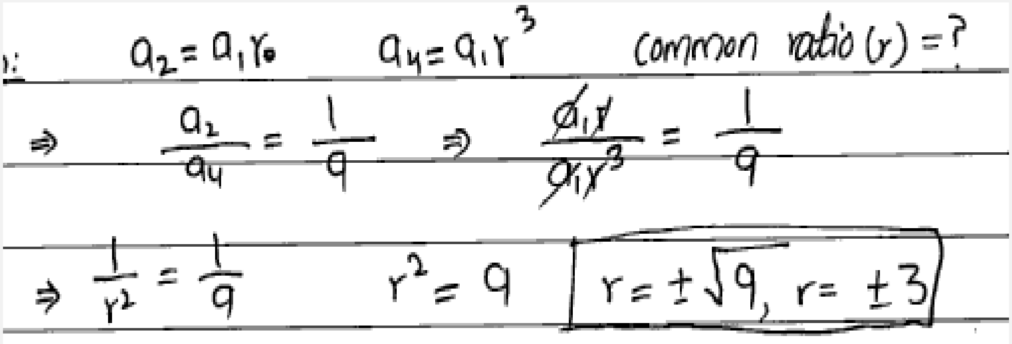
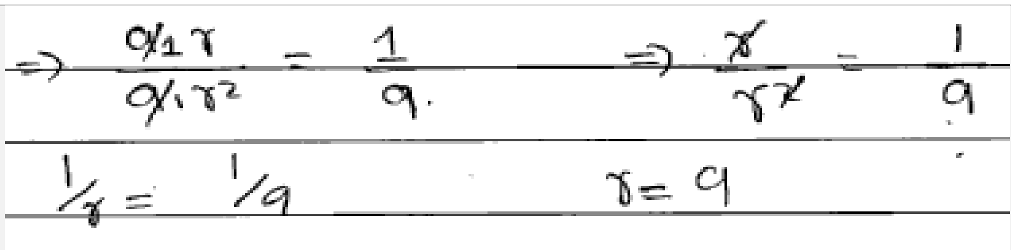
• Refer to the resource guide for extra resources		

Any Additional Suggestion:
To enhance students' understanding of determinants and their properties here are a few teaching methodologies which teachers may use.
Application-Based Projects: Assign projects that require students to apply determinants in real-world scenarios i.e., Data Analysis and Transformation Using Matrices. This allows them to see the practical relevance of the concept.
Concept Mapping: Use concept mapping or mind mapping techniques to help students connect different properties and concepts related to determinants. This visual representation can aid in understanding the relationships.
Geometric Interpretation: Explore the geometric interpretation of determinants, especially for 2D and 3D cases. Discuss how determinants are related to areas, volumes, and transformations.
Hands-On Activities: Create hands-on activities that involve physical models or interactive tools to calculate determinants. This can make the concept more tangible and engaging for students.
Interactive Software: Utilise interactive software and online tools that allow students to input matrices and calculate determinants. This provides a dynamic way for them to practice and visualise the concept.

Question No. 3i	
Question Text	The 1^{st}, 2^{nd} and 3^{rd} terms of an arithmetic sequence are 7, 12 and 17 respectively. Find the 10^{th} term.
SLO No.	3.2.3
SLO Text	Solve problems involving arithmetic sequence.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for finding the common difference. 1 mark for finding the 10^{th} term.
Overall Performance	It was observed that the majority of the candidates successfully solved the question, indicating a solid grasp of arithmetic sequences and their properties. The question was generally perceived as manageable, suggesting that candidates were proficient in applying the concepts related to arithmetic sequences to solve problems of this nature
Description of Better Responses	Candidates having better responses approached this question in two ways. Most of the candidates substituted the values in formula for the n^{th} term of the sequence and evaluated the specified term. Many candidates also applied the pattern seeking approach. In this approach, they extended the number of terms up to the specified term and found the specified term.

Image of Better Response	$a = 7 \qquad a_{10} = ? \qquad d = 12 - 7 = 5$ $a_n = a + (n-1)d$ $a_{10} = 7 + (10-1)5$ $a_{10} = 7 + (9)(5)$ $a_{10} = 7 + 45$ $a_{10} = 52.$
Description of Weaker Responses	Candidates having weaker responses substituted either incorrect values in the formula or reflected misconception in selecting the correct value of *n*, though candidate applied the correct formula. However, they were not able to substitute the correct formula for *n*.
Image of Weaker Response	1st = 7 2nd = 12 3rd = 17 $d = 5$ $a_n = a + (n-1)d$ $a_n = 10 + (7-1)5$ $a_n = 10 + (6)5$ $a_n = 10 + 30$ $= 40$ The 10th term is 52

Question No. 3ii	
Question Text	It is given that in a geometric sequence, the ratio of 2nd and 4th term is 1:9. Find the common ratio.
SLO No.	3.5.3
SLO Text	Solve problems involving geometric sequence.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for writing in the ratio form $\frac{ar^1}{ar^3} = \frac{1}{9}$. 1 mark for the cancellation and evaluating.
Overall Performance	It was observed that the majority of the candidates performed well on this question. Their ability to solve it correctly suggests a sound understanding of geometric sequences and their characteristic properties, including the common ratio. Consequently, candidates

	demonstrated competence in applying these concepts to solve problems involving geometric sequences.
Description of Better Responses	Candidates having better responses divided the 4th term with the 2nd term and equated with the given ratio in the given order. Hence, simplified and took square root to find the value of ‘*r*’.
Image of Better Response	
Description of Weaker Responses	In weaker responses, candidates began the question with correct formula and division of 4th term with the 2nd term however, they did not take square root to find the value of ‘*r*’.
Image of Weaker Response	

Question No. 3iii	
Question Text	The sum of n terms of a series is given by $S_n = \sum_{k=1}^{n}(k^2 - 1)$. Show that $S_n = \frac{n}{6}\left(2n^2 + 3n - 5\right)$.
SLO No.	4.1.3
SLO Text	Solve problems involving $\sum n$, $\sum n^2$, and $\sum n^3$.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for taking sum of n times 1 as n. 1 mark for writing in the form $S_n = \frac{n(n+1)(2n+1)}{6} - n$.
Overall Performance	Most candidates handled this question quite well. It revealed their skill in problem-solving and a solid understanding of mathematical concepts, particularly when dealing with summations. Their successful completion of the task indicated a strong grasp of mathematical principles and the ability to apply them effectively.
Description of Better Responses	In better responses, candidates expanded the expression using summation and applied the appropriate identity, took the LCM to simplify and proved the given statement.

Image of Better Response	$S_n = \sum_{k=1}^{n}(k^2-1) \Rightarrow \sum_{k=1}^{n} k^2 - \sum 1 \Rightarrow \left[\frac{n(n+1)(2n+1)}{6} - n\right]$ taking 'n' common $n\left[\frac{(n+1)(2n+1)-6}{6}\right] \Rightarrow \frac{n}{6}\left[2n^2+n+2n+1-6\right]$ $S_n \Rightarrow \frac{n}{6}\left[2n^2+3n-5\right]$ Proved
Description of Weaker Responses	In weaker responses, candidates often made errors in the second term, as they struggled to apply the relevant identity/ formula and left it unchanged from the original question. In some cases, they correctly used the formula for the first expression however, they were unable to simplify the overall expression effectively.
Images of Weaker Responses	**Image (i)** $S_n = \frac{n(n+1)(2n+1)}{6}$ $\frac{n(2n^2+n+2n+1)}{6}$ $\frac{n(2n^2+3n+5)}{6}$ $\quad \frac{n}{6}(2n^2+3n+5)$ hence Proved **Image (ii)** $\sum_{k=1}^{n}(k^2-1) = \sum_{k=1}^{n}k^2 - \sum_{k=1}^{n}1$ Answer $= \frac{n(n+1)(2n+1)}{6} - 1 \rightarrow = \frac{n}{6}(2n^2+3n-5)$ $\frac{(2n^3+3n^2+n)-6}{6} = \frac{n}{6}(2n^2+3n+1-6)$

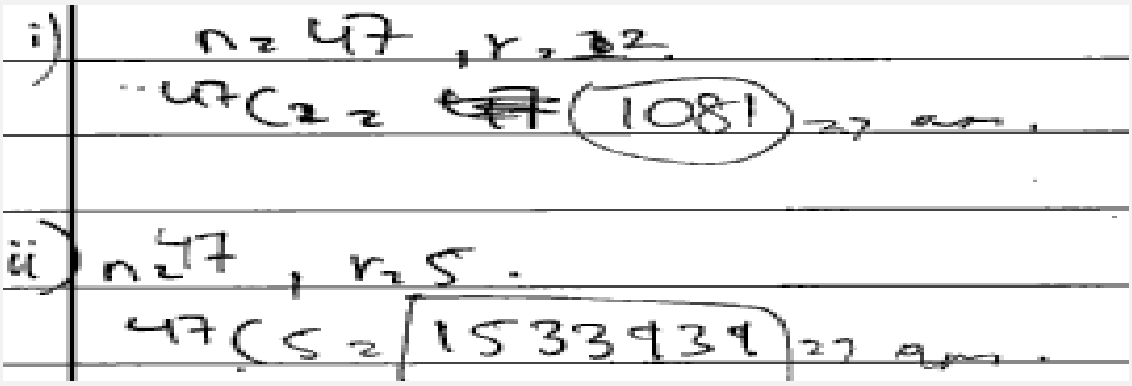
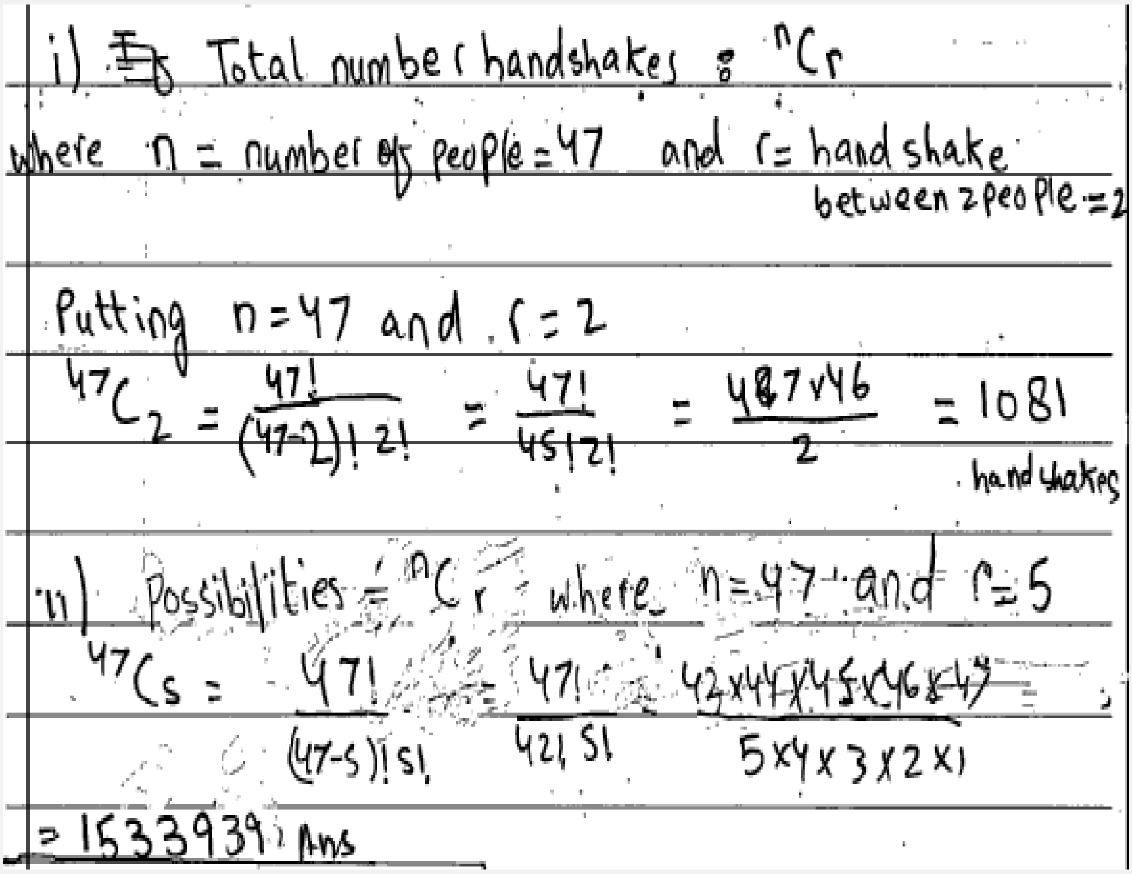
Suggestions for Improvement (Highlighted part)

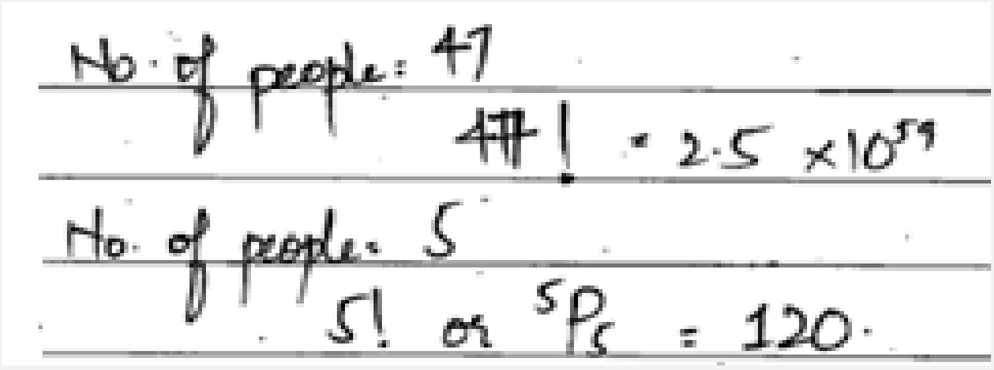
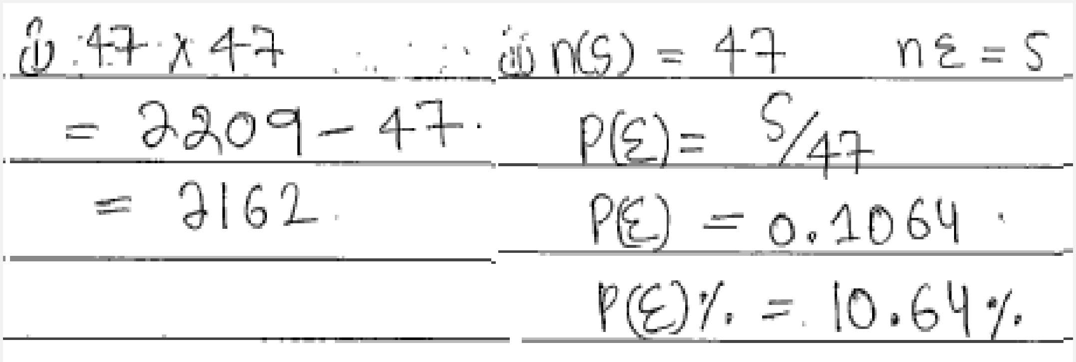
How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login
Any Additional Suggestion: To enhance students' understanding of determinants and their properties here are a few teaching methodologies which teachers may use. Online Resources: Recommend online resources, such as interactive simulations or educational videos, that supplement classroom learning and provide additional practice i.e., wolframalpha. Problem-Solving Challenges: Present students with challenging problems that require the application of the identity or formula in creative ways. Challenge them to think beyond textbook examples. Error Analysis: Analyse common errors students make when using the identity or formula. Discuss these errors and how to avoid them.		

Question No. 4i	
Question Text	On a sports day, 32 students competed in a race. The top three runners receive gold, silver, and bronze medals. Find the number of possibilities for attaining the top three positions if I. all students are present on sports day. II. two students are absent on sports day.
SLO No.	5.2.4
SLO Text	Apply ${}^{n}P_{r}$ to solve relevant problems.
Max Marks	3
Cognitive Level	A
Checking Hints	i. 1 mark for deciding between permutation and combination 1 mark for correct calculation to find the answer ii. 1 mark for correct calculation to find the answer
Overall Performance	This question involved application of permutation. Candidates struggled to identify the concept of permutation. Part (I) required the application of permutation without condition,

	while part (II) required the reduction of 2 first. Few candidates attempted this question correctly.
Description of Better Responses	Candidates with better responses applied permutation as ${}^{32}P_3$. In part (II), the condition to be applied was, the total number choices for selection was reduced by 2. Hence candidates with better responses subtracted 2 from 30 and took the permutation ${}^{30}P_3$ accurately.
Image of Better Response	I. All students => 32 Awards => 3 $32P_3$ = 29760 possibilities. II. 32-2 = 30 Now: $30P_3$ = 24360 possibilities.
Description of Weaker Responses	The number of weaker responses was quite large. Many candidates incorrectly applied fundamental principle of counting, probability and combination instead of permutation. Additionally, many candidates that applied permutation could not identify the values of *n* and *r*.
Image of Weaker Response	I. 32 students are present = ${}^{32}P_{32}$ = 2.63×10^{35} Possibilities II. two students are absent ${}^{32}P_{30}$ = 1.31×10^{35} Possibilities

Question No. 4ii	
Question Text	There were 47 people at an annual dinner of a company. I. Each of them shook hands with everyone else. Find the total number of handshakes. II. If a wall clock is given at random to 5 different people at the dinner, then find the number of possibilities for winning the gifts.
SLO No.	5.2.8
SLO Text	Solve problems involving combination.
Max Marks	3

Cognitive Level	A
Checking Hints	i. 1 mark for deciding between permutation and combination 1 mark for correct calculation to find the answer ii. 1 mark for correct calculation to find the answer
Overall Performance	Many candidates struggled to identify the combination. Only few of the candidates were able to do this question correctly.
Description of Better Responses	Part (I) required the application of combination without any condition, i.e., select any 2 from 47. Therefore, candidates with better responses applied combination as $^{47}C_2$. In part (II), total number choices for selection were the same, so better responses applied combinations as $^{47}C_5$.
Images of Better Responses	**Image (i)**  i) n = 47, r = 2 $^{47}C_2 = 1081$ ans. ii) n = 47, r = 5. $^{47}C_5 = 1533939$ ans. **Image (ii)**  i) Total number handshakes = ^nC_r where n = number of people = 47 and r = hand shake between 2 people = 2 Putting n = 47 and r = 2 $^{47}C_2 = \frac{47!}{(47-2)!2!} = \frac{47!}{45!2!} = \frac{47\times46}{2} = 1081$ handshakes ii) Possibilities = ^nC_r where n = 47 and r = 5 $^{47}C_5 = \frac{47!}{(47-5)!5!} = \frac{47!}{42!5!} = \frac{43\times44\times45\times46\times47}{5\times4\times3\times2\times1}$ = 1533939 Ans

Description of Weaker Responses	In weaker responses, candidates used permutations and fundamental counting concepts. However, they struggled to accurately identify the appropriate concept. Additionally, they applied other methods such as probability, multiplication, squaring, and division which were not applicable to solve this question.
Images of Weaker Response	**Image (i)**  **Image (ii)** 

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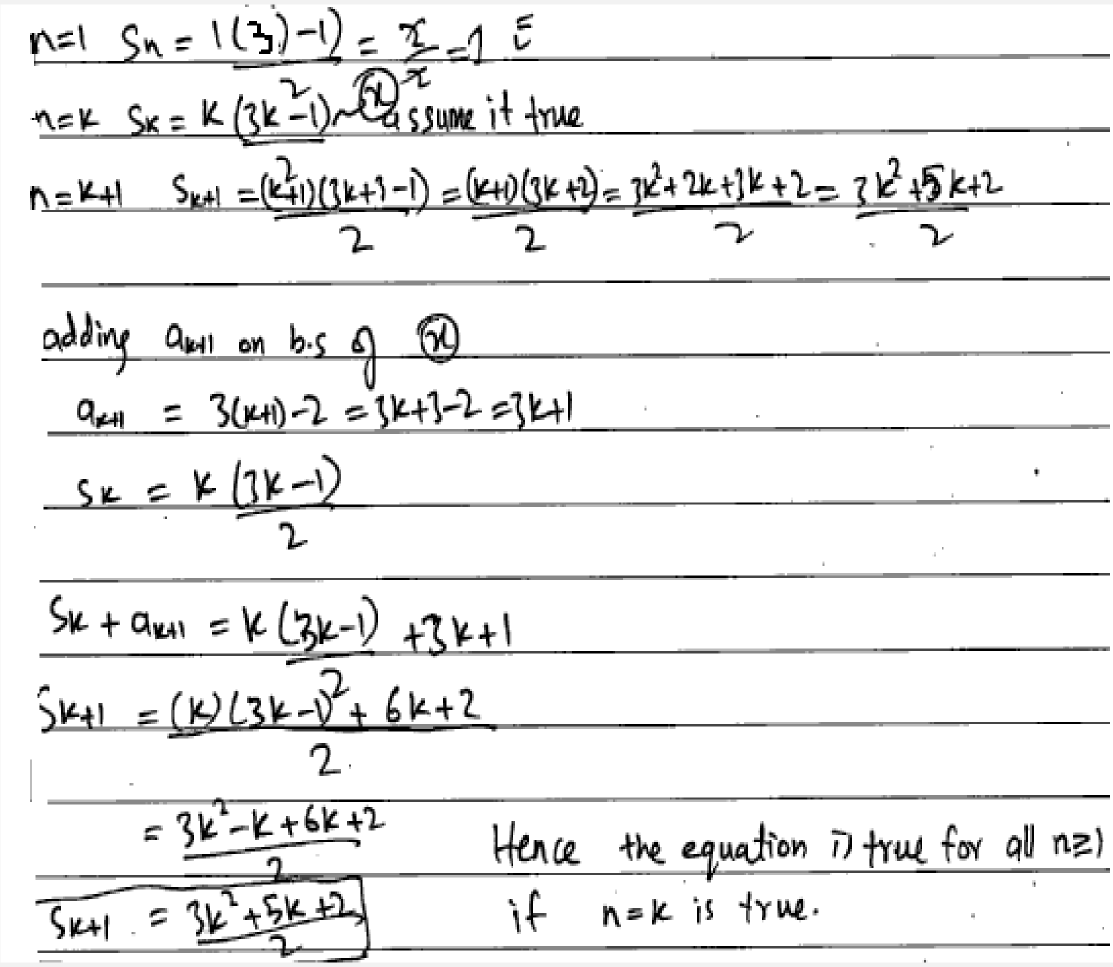
Any Additional Suggestion:
By using these teaching methodologies, teachers can help students grasp the distinctions between permutations and combinations. Hands-On Activities: Engage students in tangible activities using objects like coins, cards, or building blocks. Let them physically arrange these objects to understand permutations. For example, ask them to arrange five coloured balls in different orders. Interactive Simulations: Use online tools and simulations that allow students to experiment with permutations. There are various permutation calculators and apps available that can help students practice and visualise permutations. Combinations vs. Permutations: Clearly differentiate between permutations and combinations. Use examples to show when each concept is appropriate. This distinction is essential to prevent confusion. Storytelling: Create stories or scenarios that involve arranging items in different orders. This can make the concept more relatable and interesting for students. Scaffolding: Break down complex permutation problems into smaller, manageable steps. This helps students understand the sequential nature of permutations and how to approach problems systematically. Real-World Applications: Illustrate how permutations are used in real-life situations, such as password combinations, seating arrangements, and lock combinations. Show the practical significance of permutations. Error Analysis: Encourage students to analyse their mistakes when solving permutation problems. Understanding common errors can help them improve their problem-solving skills.

Question No. 5i	
Question Text	Using mathematical induction, prove that for $n \geq 1$, $1+4+7+(3n-2)=\dfrac{n(3n-1)}{2}$.
SLO No.	6.1.2
SLO Text	Prove the statements, identities and formulae using the principle of mathematical induction. (**Note:** Questions involving inequalities are not included for example: $n^2 > n + 3$ for integral values of $n \geq 3$).
Max Marks	4
Cognitive Level	A
Checking Hints	1 mark for finding proving for $S(1)$ 1 mark for assuming for S(k) 1 mark for adding $(k + 1)^{th}$ term on both sides 1 mark for reaching the required result (proof)
Overall Performance	Overall, it can be observed that candidates' performance on this question was diverse. Some displayed a clear understanding of mathematical induction, while others faced challenges, particularly when it came to extending the proof to the $(k + 1)^{th}$ term.
Description of Better Responses	The better responses showed that candidates applied all the three conditions and obtained the required conclusion. The candidates with such responses proved that statement for $n = 1$, then assumed it for $n = k$, added the $(k + 1)^{th}$ term on both sides and obtained the statement for $(k + 1)^{th}$ term.

<table>
<tr><td>Image of Better Response</td><td>

$n=1 \quad S_n = \frac{1(3-1)}{2} = \frac{2}{2} = 1$

$n=k \quad S_k = \frac{k(3k-1)}{2}$ (x) assume it true

$n=k+1 \quad S_{k+1} = \frac{(k+1)(3k+3-1)}{2} = \frac{(k+1)(3k+2)}{2} = \frac{3k^2+2k+3k+2}{2} = \frac{3k^2+5k+2}{2}$

adding a_{k+1} on b.s of (x)

$a_{k+1} = 3(k+1)-2 = 3k+3-2 = 3k+1$

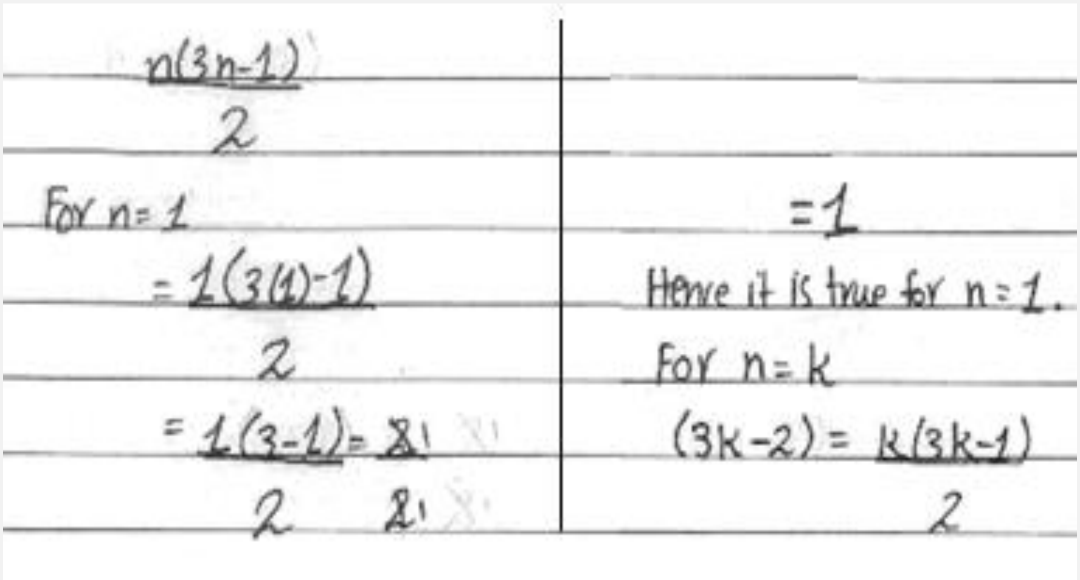
$S_k = \frac{k(3k-1)}{2}$

$S_k + a_{k+1} = \frac{k(3k-1)}{2} + 3k+1$

$S_{k+1} = \frac{(k)(3k-1) + 6k+2}{2}$

$= \frac{3k^2-k+6k+2}{2}$

$\boxed{S_{k+1} = \frac{3k^2+5k+2}{2}}$

Hence the equation is true for all $n \geq 1$ if $n=k$ is true.</td></tr>
<tr><td>Description of Weaker Responses</td><td>In weaker responses, candidates were unable to prove the given statement. Such responses vary in terms of solutions. Some candidates stopped solving after applying $n = 1$. Those who tried to further work it out could not obtain the required result. Hence, such responses could not conclude whether the statement is correct or not.</td></tr>
<tr><td>Image of Weaker Response</td><td>

$\frac{n(3n-1)}{2}$

For $n=1$

$= \frac{1(3(1)-1)}{2}$

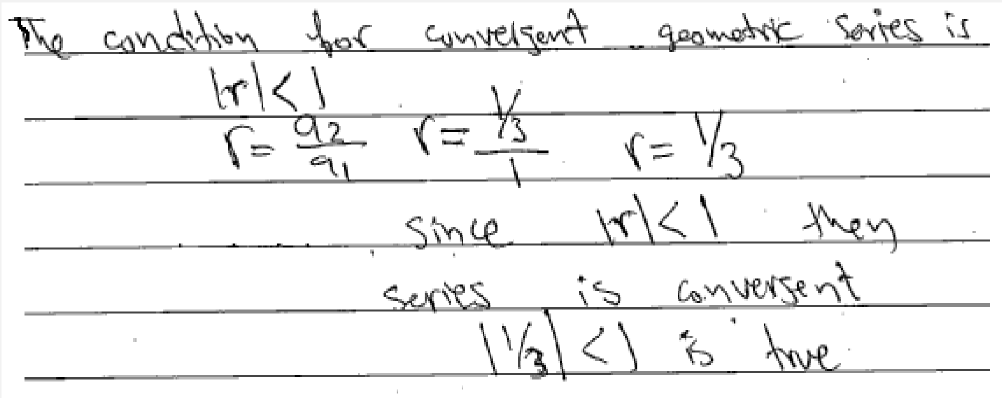
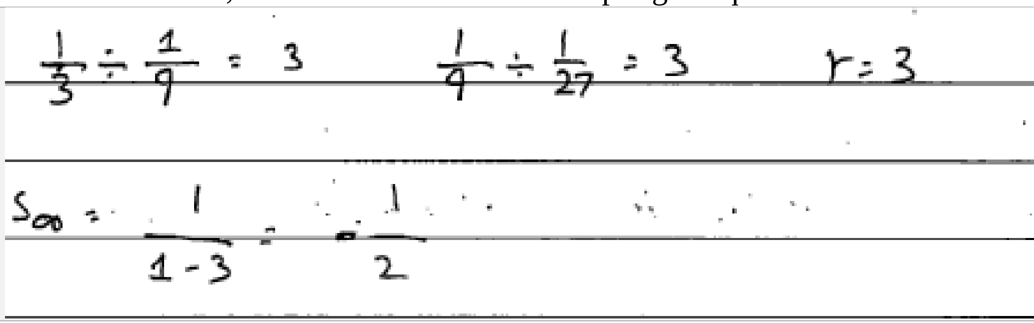
$= \frac{1(3-1)}{2} = \frac{2}{2}$

$=1$

Hence it is true for $n=1$.

For $n=k$

$(3k-2) = \frac{k(3k-1)}{2}$</td></tr>
</table>

Question No. 5ii	
Question Text	Show that the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+....$ is a convergent geometric series.
SLO No.	6.3.3
SLO Text	Explain the convergence of $(x+y)^n$ for $\lvert x\rvert<1$ where n is a rational number.
Max Marks	2
Cognitive Level	U
Checking Hints	1 mark for finding calculating the value of r. 1 mark for stating the nature of the sequence based on the value of $\lvert r\rvert$.
Overall Performance	This question was related to convergence of $(x+y)^n$ for $\lvert x\rvert<1$, where n is a rational number. The overall performance of candidates in this question, which focused on determining the convergence of a series involving the parameter x (or r), displayed a notable variation.
Description of Better Responses	The better responses accurately calculated the value of x (or r as stated). They began by computing and evaluating the 'r' value using the ratio of 2^{nd} and 1^{st} terms, gave the reason $\lvert r\rvert<1$ and determined that the series was convergence.
Image of Better Response	
Description of Weaker Responses	In weaker responses, some of candidates applied the formula to find the sum of infinite terms, while others did not attempt this question. Hence, the former lost marks due to incorrect solution, while latter due to not attempting the question.
Image of Weaker Response	

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words	• Story Board • Cause and Effect • Fish and Bone	• Past paper questions • Discussion on E-Marking Notes

• Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login
Any Additional Suggestion: By implementing these teaching methodologies, teachers can help students develop a stronger understanding of mathematical induction, improve their problem-solving skills, and reduce common mistakes. Problem-Solving Approach: Encourage students to think critically when faced with problems. Teach them to analyse the problem statement, identify the given information, and plan a logical approach to prove or disprove a statement. Multiple Approaches: Highlight that there can be multiple approaches to solving a problem. Encourage students to explore different methods and approaches before settling on a solution. Error Analysis: Discuss common mistakes made by candidates in weaker responses, such as prematurely stopping calculations or avoiding questions altogether. Analyse these mistakes as a class and discuss how to avoid them. Practice Problems: Provide students with a range of problems related to the convergence of rational numbers. Encourage them to explore patterns and make conjectures.		

Question No. 6a Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.	
Question Text	The relationship between the roots of a quadratic equation are given such that $3\alpha - \beta = 7$ and $\alpha = \frac{\beta}{2} + 1$, where α and β are the roots of a quadratic equation. Find the i. values of α and β. ii. values of $\alpha + \beta$ and $\alpha\beta$. iii. quadratic equation.
SLO No.	7.5.4d
SLO Text	Find the value(s) of unknown(s) involved in a given quadratic equation when d) roots satisfy a given relation. (e.g. the relation $2\alpha + 5\beta = 7$ and $\alpha = \beta$, where α and β are the roots of given equation).
Max Marks	6
Cognitive Level	A
Checking Hints	i. 1 mark for substituting α or β in the equation. 1 mark for the values of α. 1 mark for the values of β.

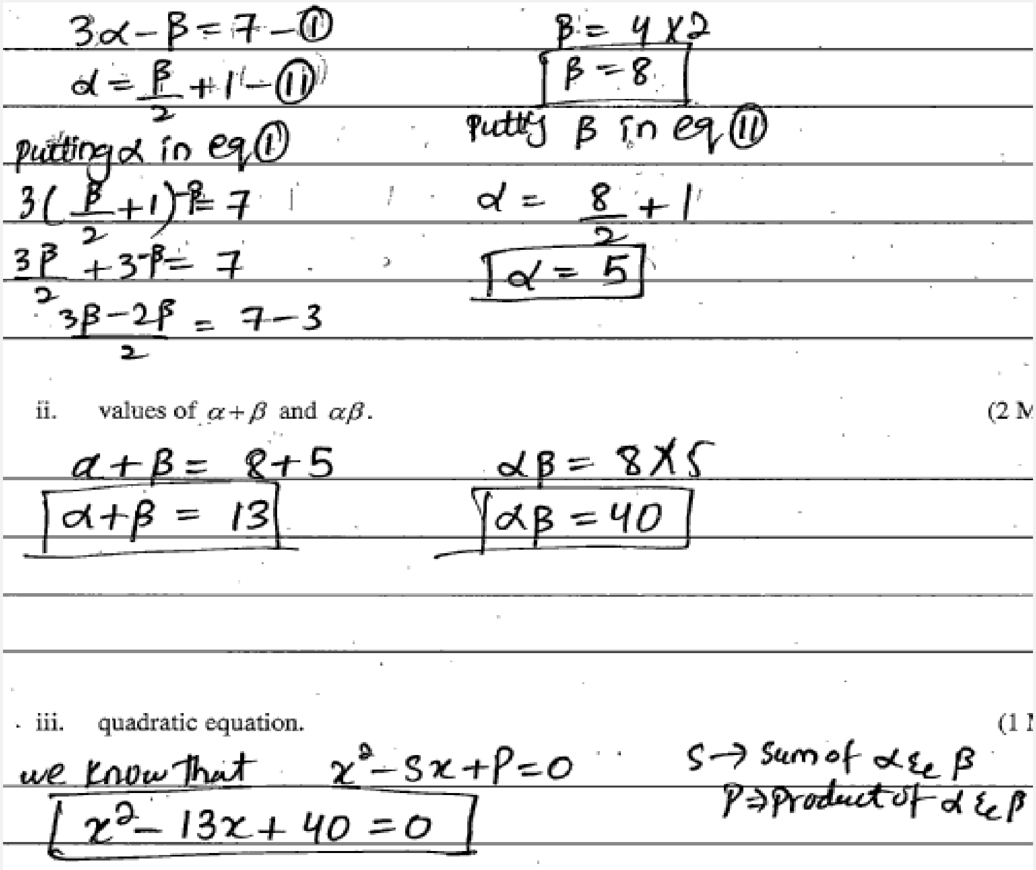
	ii. 1 mark for finding $\alpha+\beta$ 1 mark for finding $\alpha\beta$ 1 mark for the equation
Overall Performance	The question appeared challenging to most candidates, with only a handful managing to meet its requirements by correctly finding the root values and formulating the necessary equation. These successful candidates demonstrated a solid understanding of both the quadratic equation and its roots. However, a significant number of candidates did not reach the expected level of comprehension.
Description of Better Responses	Better responses followed a thorough process. They used the elimination method to find values for α and β accurately and used it for the sum and product of the roots, and derived the correct quadratic equation.
Image of Better Response	 $3\alpha-\beta=7$ — (i) $\alpha=\frac{\beta}{2}+1$ — (ii) putting α in eq (i) $3\left(\frac{\beta}{2}+1\right)-\beta=7$ $\frac{3\beta}{2}+3-\beta=7$ $\frac{3\beta-2\beta}{2}=7-3$ $\beta=4\times2$ $\boxed{\beta=8}$ putting β in eq (ii) $\alpha=\frac{8}{2}+1$ $\boxed{\alpha=5}$ ii. values of $\alpha+\beta$ and $\alpha\beta$. (2 M $\alpha+\beta=8+5$ $\alpha\beta=8\times5$ $\boxed{\alpha+\beta=13}$ $\boxed{\alpha\beta=40}$ iii. quadratic equation. (1 M we know that $x^2-Sx+P=0$ $S\rightarrow$ Sum of α & β $P\Rightarrow$ Product of α & β $\boxed{x^2-13x+40=0}$
Description of Weaker Responses	Weaker responses from candidates attempting the question indicated the following reasons for their challenges: Neglecting β Calculation: Weaker candidates often failed to calculate the value of β, a vital element in solving the problem. Inaccurate Sum and Product of Roots: Candidates exhibited an additional error by erroneously computing the sum and product of roots. Instead of using the correct values for α and β, they substituted inappropriate values like $-\beta/3$ and $-7/3$, leading to an incorrect formulation of the quadratic equation.

Image of Weaker Response	$3\alpha - \beta = 7$ $\quad\quad$ $3\alpha - \beta = 7$ $3\alpha = 7 + \beta$ $\quad\quad$ $3\alpha - 7 = \beta$ $\alpha = \frac{7 + \beta}{3}$ values of $\alpha + \beta$ and $\alpha\beta$. $\alpha + \beta = \frac{-b}{a}$ $\boxed{\frac{-\beta}{3}}$ Ans $\quad\quad$ $\therefore 3\alpha - \beta - 7 = 0$ $\alpha \cdot \beta = \frac{c}{a} = \boxed{\frac{-7}{3}}$ Ans. i. quadratic equation. $3\alpha - \beta - 7 = 0$

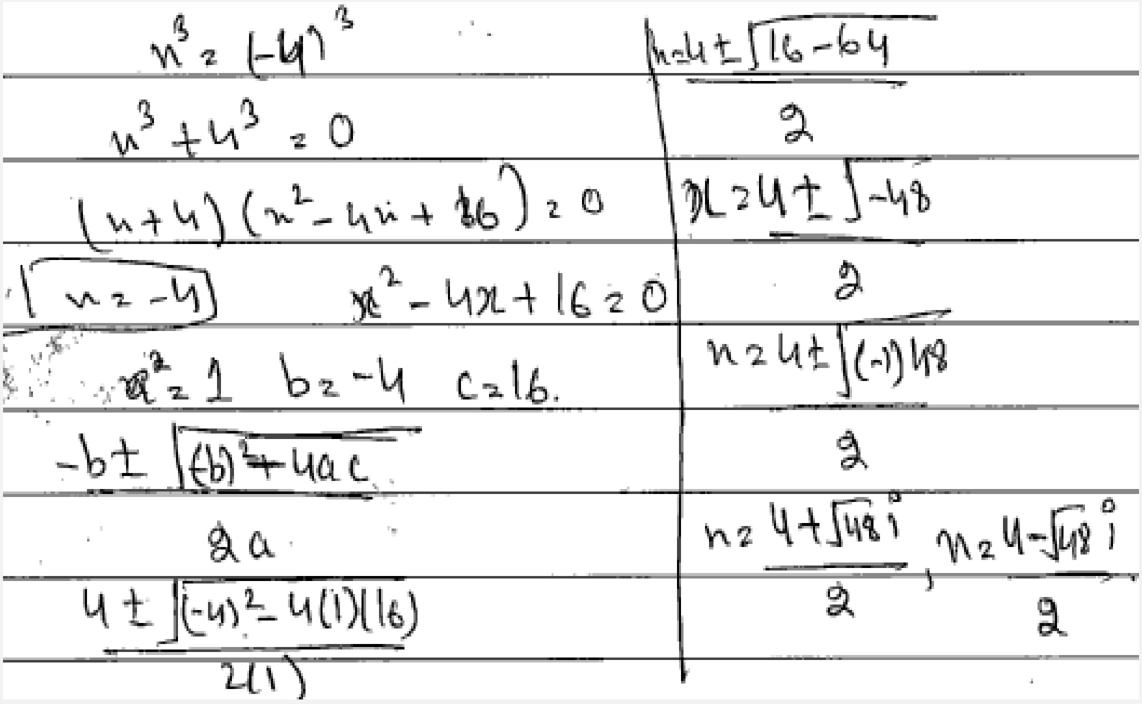
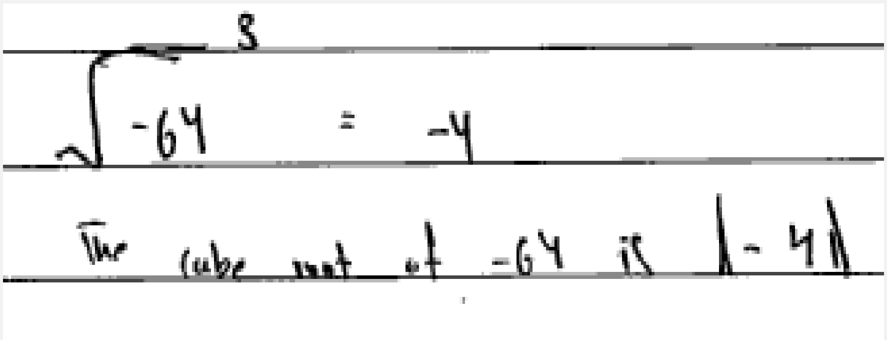
Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
Following are some teaching activities that teacher may use, these activities can make learning about quadratic equations more dynamic and interactive, catering to diverse learning styles and fostering a deeper understanding of the topic.

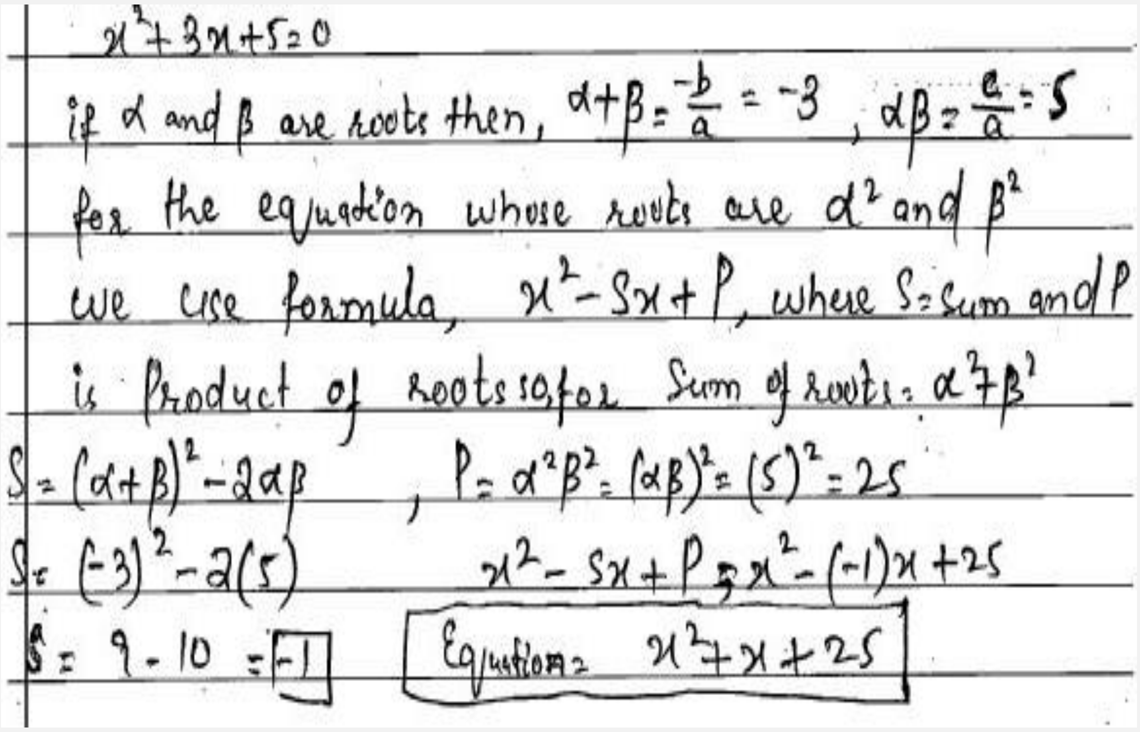
Interactive Simulations: Utilise educational technology such as interactive software or online platforms and interactive online simulations or apps to provide additional practice and resources for students to reinforce their learning. that allow students to manipulate the parameters of quadratic equations and observe how changes affect the graph and solutions.
Real-World Problem Solving: Provide students with real-world scenarios that can be modelled using quadratic equations (e.g., projectile motion, profit optimisation). Let them formulate and solve the corresponding equations.
Encourage Critical Thinking: Encourage students to think critically when solving quadratic equations. Teach them to analyse problems, identify the most appropriate method to use, and validate their solutions.
Error Analysis: Analyse common mistakes made by candidates in weaker responses, such as neglecting to calculate β and using incorrect values for the sum and product of roots. Discuss these errors with students, emphasising the importance of accuracy.

Question No. 6bi Candidates were given the choice to attempt any ONE out of the two questions: 6a and 6b.	
Question Text	Find the cube roots of -64.
SLO No.	7.4.2
SLO Text	Find the cube roots of unity and other numbers (e.g. ± 8, ± 27 etc).
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for writing $x^3 + 64 = 0$ 1 mark for factorisation to get $\left(x+4\right)\left(x^2 - 4x + 16\right) = 0$ 1 mark for application of quadratic formula to get $x = -4 \quad or \quad x = \dfrac{4 \pm \sqrt{16 - 4 \times 1 \times 16}}{2}$
Overall Performance	Majority of the candidates selected this question. The performance in this section can be considered quite satisfactory, as the majority of candidates successfully tackled the question. They adeptly identified and computed the cube roots of unity and other designated numbers, showcasing a solid grasp of the topic and precise calculation skills. However, there were some weaker responses that shed light on candidates' struggles when it came to effectively solving cubic equations.
Description of Better Responses	Better responses showed all the essential steps to find cube roots of −64. Such responses started with $x^3 + 64 = 0$ and found real and complex roots by applying quadratic formula and were able to find all three roots.

Image of Better Response	
Description of Weaker Responses	Weaker responses showed candidates' inability to solve the cubic equation. Such candidates were unable to find all three roots and wrote only one root – 4 by taking the cubic root of – 64 directly and did not continue further for other two roots.
Image of Weaker Response	

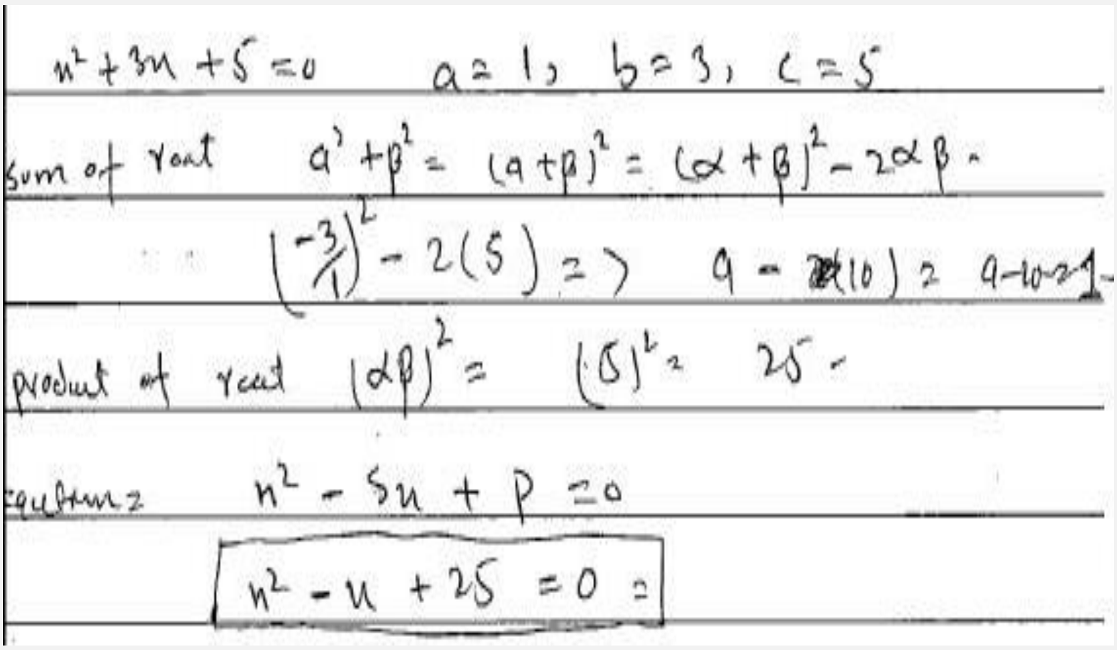
Question No. 6bii	
Question Text	If α and β are the roots of the quadratic equation $x^2 + 3x + 5 = 0$, then find the quadratic equation whose roots are α^2 and β^2.
SLO No.	7.6.3b
SLO Text	Find a quadratic equation whose root are ii. α^2, β^2 .
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for finding sum and product of the roots for the given equation. 1 mark for finding sum and product of root of the required equation. 1 mark for finding the required equation.

<table>
<tr><td>Overall Performance</td><td>This question 6b(ii) was related to finding quadratic equation using roots. Most of the candidates attempted this question correctly. However, some candidates could not perform well in this question because they did not find sum of roots correctly.</td></tr>
<tr><td>Description of Better Responses</td><td>Better responses, demonstrated all necessary steps. They first determined the formula for sum and product of roots as $(\alpha+\beta)^2-2\alpha\,\beta^2$ and $(\alpha\,\beta)^2$ and then made quadratic equation by substituting sum and product of the required roots.</td></tr>
<tr><td>Images of Better Responses</td><td>Image (i)



$x^2+3x+5=0$
if α and β are roots then, $\alpha+\beta=\frac{-b}{a}=-3$, $\alpha\beta=\frac{c}{a}=5$
for the equation whose roots are α^2 and β^2
we use formula, x^2-Sx+P, where S = sum and P
is Product of roots so, for Sum of roots = $\alpha^2+\beta^2$
$S=(\alpha+\beta)^2-2\alpha\beta$, $P=\alpha^2\beta^2=(\alpha\beta)^2=(5)^2=25$
$S=(-3)^2-2(5)$ $x^2-Sx+P=x^2-(-1)x+25$
$S=9-10=\boxed{-1}$ $\boxed{\text{Equation} = x^2+x+25}$

Image (ii)



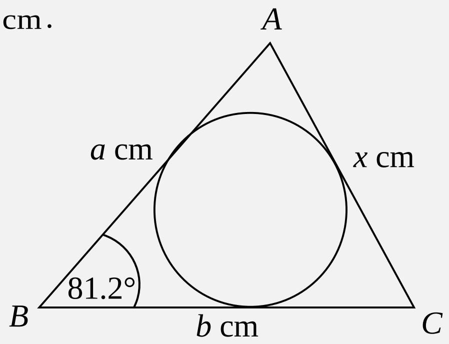
$x^2+3x+5=0$ $a=1,\ b=3,\ c=5$
sum of root $\alpha^2+\beta^2=(\alpha+\beta)^2=(\alpha+\beta)^2-2\alpha\beta$ -
$\left(\frac{-3}{1}\right)^2-2(5)\Rightarrow 9-(10)=9-10=-1$ -
product of root $(\alpha\beta)^2=(5)^2=25$ -
equation = $x^2-Sx+P=0$
$\boxed{x^2-x+25=0}$
</td></tr>
</table>

Description of Weaker Responses	In weaker responses, most of the candidates did not make the formula for sum of the required roots properly. They took the square of $(\alpha + \beta)^2 = (3)^2 = 9$. However, they calculated product of the required roots correctly and made the quadratic equation from it.
Images of Weaker Response	**Image (i)** α and $\beta = x^2+3x+5=0$ α^2 and $\beta^2 = x^2+9x+25=0$ The equation for α^2 and β^2 is $= x^2+9x+25=0$ **Image (ii)** $x^2 - \alpha^2+\beta^2 + (\alpha\beta)^2 = 0$ $x^2 - (\alpha+\beta)^2 x + \left(\frac{c}{a}\right)^2 = 0$ $x^2 - \left(\frac{-b}{a}\right)^2 x + \left(\frac{5}{1}\right)^2 = 0$ $x^2 - (9)x + 25 = 0$ $x^2 + 9x + 25 = 0$

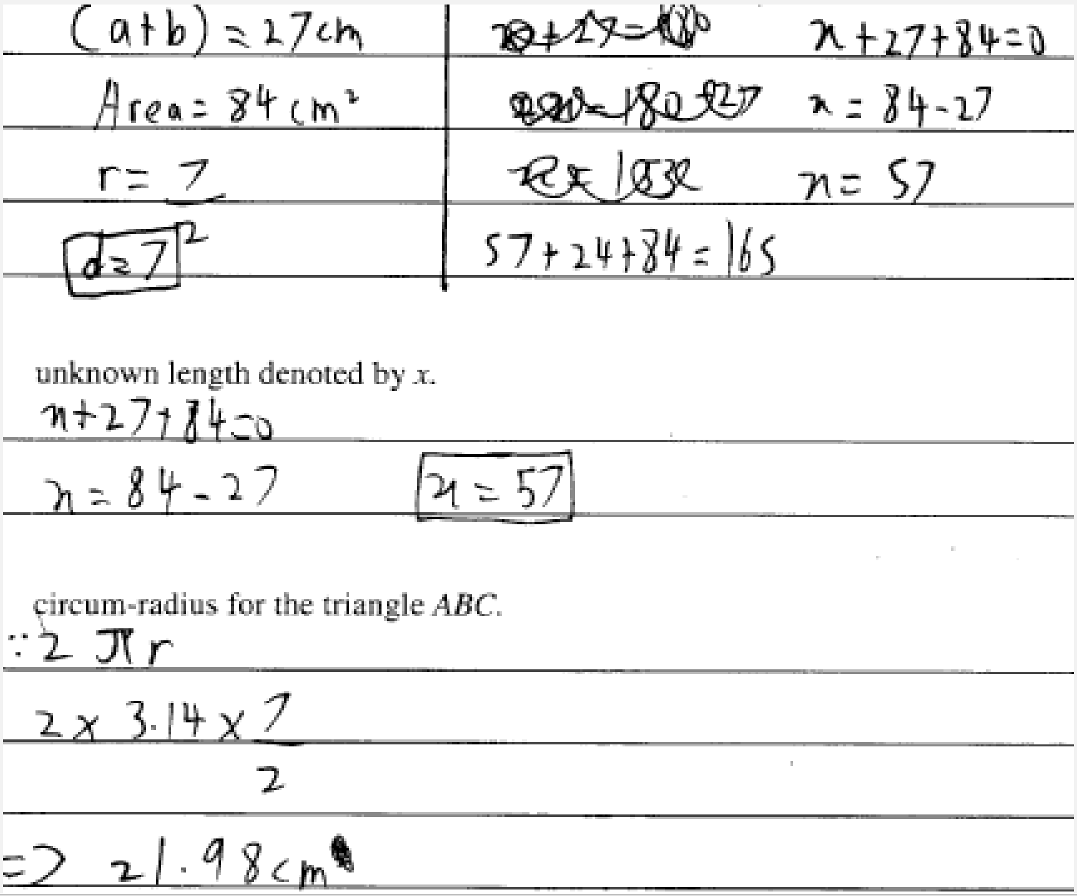
Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Refer to the resource guide for extra resources		
Any Additional Suggestion: Following are some recommendations and online tools that teachers can utilise to enhance their teaching of this concept, providing students with a more effective and comprehensive understanding. Present equations involving cube roots of unity and guide students through the process of solving them. Show them how to express complex solutions in terms of ω and its powers. Provide examples and practice problems that involve cube roots of unity, in algebraic manipulations. Encourage students to work through these problems step by step. Utilise online math visualisation tools like GeoGebra or Desmos to plot the complex cube roots of unity on the complex plane. Students can see how these roots are distributed and explore their geometric properties. Show how cube roots of unity are equally spaced around the unit circle. Websites like Wolfram Alpha can be helpful. Explore online graphing calculators that allow students to input complex numbers and visualise the cube roots and other roots in graphical form. Symbolab ise useful tools for this purpose.		

Question No. 7a Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.	
Question Text	• Consider the given diagram. $(a+b)$ cm $= 27$ cm. • Area $= \Delta = 84$ cm^2. • Radius of inscribed circle is $\frac{7}{2}$ cm. Using the given information, calculate the i. the perimeter of the triangle. ii. the unknown length denoted by *x*. iii. the circum-radius for the triangle *ABC*. 
SLO No.	9.3.3
SLO Text	Find the circum-radius, in-radius and escribed radii using the formulae mentioned in SLO 9.3.2.
Max Marks	5
Cognitive Level	A
Checking Hints	i. 1 mark for writing substituting the values in the formula for in-radius. 1 mark for finding the perimeter as 2*s*. ii. 1 mark for writing finding the value of *x*. iii. 1 mark for identifying circum-radius. 1 mark for substitution in the formula and evaluation.
Overall Performance	The question's performance exhibited variation among candidates. Only few of the candidates demonstrated a strong understanding of the concept, yielding correct solutions. However, many responses highlighted candidates' struggles in accurately applying the concept.

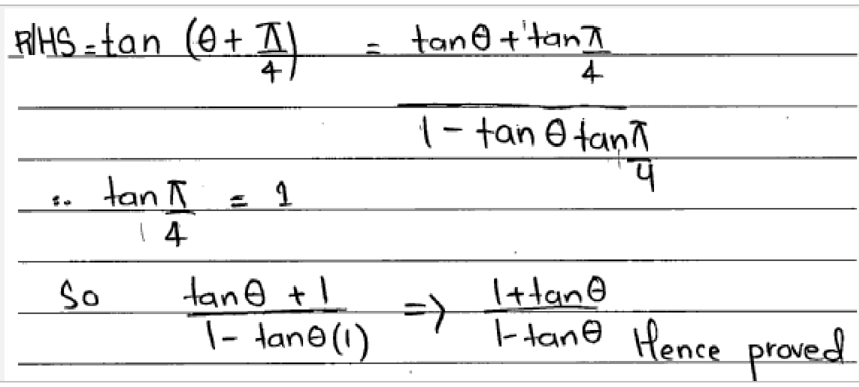
Description of Better Responses	Better responses showed all the essential steps to find the required elements. Such responses applied the correct formula of in-radius to find the perimeter of the triangle first, then applied the appropriate formula to get the circum-radius of the triangle.
Image of Better Response	$r = \frac{\Delta}{s}$ $\quad$ $7s = 84 \times 2$ $\quad$ $s = \frac{a+b+c}{2}$ $\frac{7}{2} \cancel{=} \frac{84}{s}$ $\quad$ $7s = 168$ $s = \frac{168}{7} = 24$ $\quad$ $24 \times 2 = a+b+x$ (Perimeter) $48 = a+b+x$ ii. unknown length denoted by *x*. (1 M $\because a+b+x = 48cm$ $\quad$ $27+x = 48$ $\quad$ $x = 21$ cm $(a+b) = 27$ cm $\quad$ $x = 48-27$ iii. circum-radius for the triangle *ABC*. (2 Ma $R = \cancel{\frac{abc}{4\Delta}} \frac{b}{2\sin\beta}$ $R = \frac{21}{2\sin(81.2^\circ)} = \frac{21}{1.98} = \boxed{10.62\ cm}$
Description of Weaker Responses	The weaker responses reflected the candidate's inability to apply the concept. For example, certain responses showed that candidates applied the concept of probability instead of applying the concept of in-circle, radii and area of a triangle. While some candidates found incorrect diameter from the given in-radius which was not required, such responses calculated the length of the required side of the tringle by subtracting area's and two side lengths. Additionally, they applied incorrect formula of circumference of the circle to calculate circumradius.
Images of Weaker Responses	**Image (i)** $P(A \cup B) = P(A) \times P(B)$ $\frac{27}{84} = \frac{9}{28}$ $\frac{9}{28} \div \frac{7}{2} = \frac{9}{392}$ $81.2^\circ \times 81.2^\circ \times 17.6$ $= 11604.544$

	Image (ii) 

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
By implementing these strategies, teachers can help students overcome the identified mistakes and improve their problem-solving skills in mathematics, particularly when dealing with concepts related to in-circle, radii, and area of a triangle.
Practice with Similar Problems: Provide additional practice problems that involve in-circle, radii, and area of a triangle to reinforce the correct application of this concept.
Diameter Awareness: Highlight that finding the diameter from the given in-radius is not required for solving the problem. Encourage students to focus on the essential steps.
Calculation Check: Emphasise the importance of double-checking calculations, especially when calculating lengths and circumradii. Encourage students to re-evaluate their solutions for accuracy.
Error Analysis: Analyse common errors made by candidates in their responses and discuss these mistakes as part of the learning process. Provide guidance on how to avoid these errors in future problem-solving.

Question No. 7b Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.	
Question Text	Show that i. $\tan\left(\theta+\frac{\pi}{4}\right)=\frac{1+\tan\theta}{1-\tan\theta}$. ii. $\frac{\sin\theta}{\sin 2\theta}+\frac{\sin 2\theta}{\cos\theta}=\frac{1+2\sin 2\theta}{2\cos\theta}$.
SLO No.	8.3.4
SLO Text	Solve problems related to fundamental law of trigonometry and its deductions.
Max Marks	5
Cognitive Level	A
Checking Hints	i. 1 mark for expanding using the relevant identity. 1 mark for putting the known angle and simplify to get the RHS. ii. 1 mark for substituting for $\sin 2\theta$. 1 mark for cancellations and retaking LCM for addition. 1 mark for writing in the form of 2sin2θ.
Overall Performance	Question 7b(i) was related to application of trigonometric identities, particularly involving sum of angles. Majority of the candidates attempted this question correctly.
Description of Better Responses	Better responses exhibited thorough knowledge and application of trigonometric identities. The candidates expand the left-hand side by correct identity of $\tan(\alpha+\beta)$ and substituted correct value of tan $\frac{\pi}{4}$ and successfully obtain the right-hand side.
Image of Better Response	 RHS = $\tan\left(\theta+\frac{\pi}{4}\right) = \frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$ $\therefore \tan\frac{\pi}{4} = 1$ So $\frac{\tan\theta+1}{1-\tan\theta(1)} \Rightarrow \frac{1+\tan\theta}{1-\tan\theta}$ Hence proved

Description of Weaker Responses	Weaker responses, represented confusion in trigonometric identities. A large number of candidates did not apply the correct identity to expand the left-hand side and wrote $\tan\theta + \tan\frac{\pi}{4}$ directly. Also, many candidates took the LCM of the angle that led them to incorrect answers.
Image of Weaker Response	$\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1+\tan\theta}{1-\tan\theta}$ $\tan\left[\frac{4\theta + \pi}{4}\right] = \frac{1+\tan\theta}{1-\tan\theta}$ $\tan\left[\frac{4\theta - 180}{4}\right] = \frac{1+\tan\theta}{1-\tan\theta}$ $\cancel{\tan}\ \frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\tan\theta}{1-\tan\theta}$
Overall Performance	Question 7b(ii) was related to application of trigonometric identities. Most of the candidates attempted this question correctly.
Description of Better Responses	Better responses showed all the essential steps to prove the given expression. Such candidates took the LCM first; then, simplified the expression and applied the correct trigonometric identity of sin2 θ to successfully obtain the required result.
Image of Better Response	$\frac{\sin\theta}{2\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{\cos\theta}$ $\Rightarrow \frac{\sin\theta + 2\sin\theta\cos\theta(2\sin\theta)}{\cos\theta(2\sin\theta)}$ $\Rightarrow \frac{\cancel{\sin}\theta(1+4\sin\theta\cos\theta)}{\cos\theta(2\cancel{\sin}\theta)}$ $= \frac{1+4\sin\theta\cos\theta}{2\cos\theta}$ $= \frac{1+2(2\sin\theta\cos\theta)}{2\cos\theta}$ $= \frac{1+2\sin 2\theta}{2\cos\theta}$ = LHS = RHS = proved
Description of Weaker Responses	In weaker responses, difficulties arose when attempting to simplify trigonometric functions, and these candidates were unable to recognise the necessary steps for proving the result. Additionally, candidates made incorrect simplifications and employed the wrong LCM, which highlighted their confusion in trigonometric simplification.

Image of Weaker Response	$\frac{\cancel{\sin\theta}}{\cancel{\sin 2\theta}} + \frac{\cancel{\sin 2\theta}}{\cos\theta} = \frac{1+2\sin 2\theta}{2\cos\theta}$ $\frac{\sin\theta}{1} + \frac{1}{\cos\theta} = \frac{1+2\sin 2\theta}{2\cos\theta}$ $\frac{\sin\theta}{\cos\theta} = \frac{1+2\sin 2\theta}{2\cos\theta}$ $\tan\theta = 1+2\tan 2\theta$

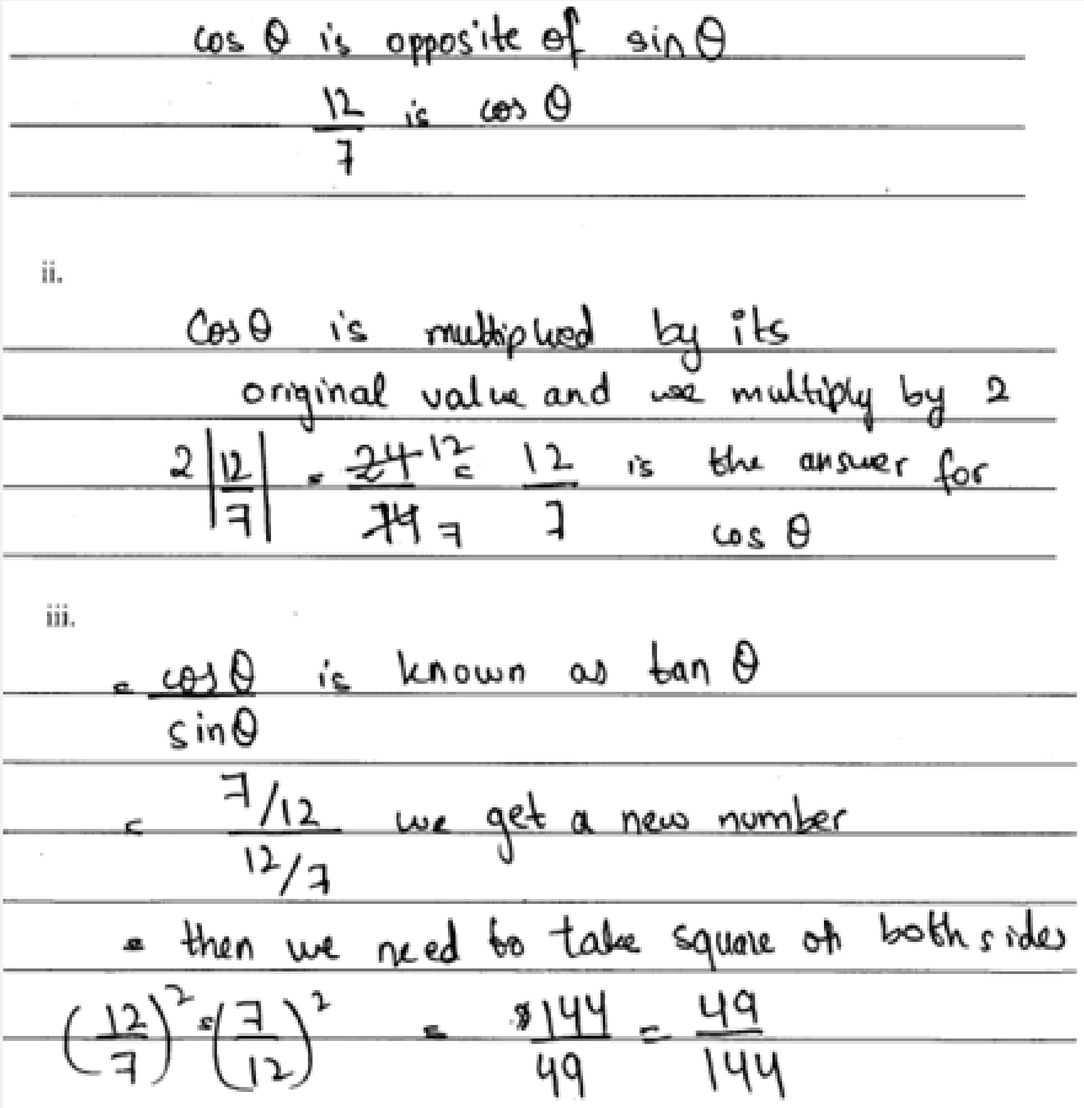
Suggestions for Improvement (Highlighted part).

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
Following are some approaches to help students understand and apply these concepts.
Introduce proofs of trigonometric identities step by step. Start with simple identities and gradually progress to more complex ones. Encourage students to derive identities themselves.
Encourage group discussions and problem-solving sessions where students work together to prove or apply trigonometric identities. Peer teaching can reinforce understanding.
Use interactive quizzes and self-assessment tools that allow students to practice verifying and applying trigonometric identities. Immediate feedback is valuable for learning.
Ask students to create concept maps or diagrams that illustrate the relationships between trigonometric identities, such as the reciprocal, quotient etc.

Question No. 7c Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b and 7c.	
Question Text	A terminal ray in the first quadrant makes an angle θ with the x-axis. If $\sin\theta = \frac{7}{12}$, then find i. $\cos\theta$. ii. $\cos 2\theta$. iii. $\tan\theta$.
SLO No.	8.1.3
SLO Text	Find the values of remaining trigonometric ratios if one of the trigonometric ratios is given.
Max Marks	5
Cognitive Level	A
Checking Hints	i. 1 mark for using appropriate identity. 1 mark for substituting the values. ii. 1 mark for using the appropriate identity. 1 mark for substitution. iii. I mark for finding tangent of the angle.
Overall Performance	Question 7c was related to finding trigonometric ratios. Most of the candidates attempted this question correctly using correct identities and conversions.
Description of Better Responses	Better responses analysed that terminal of the angle was in first quadrant and applied relevant trigonometric identities i.e., $\sin^2\theta + \cos^2\theta$=1 and $\cos 2\theta = \cos^2\theta - \sin^2\theta$, substituted the given identities to find the unknown values successfully.
Image of Better Response	i. cosθ. $\because$ All function +ve in (I) quadrant $\sin^2\theta + \cos^2\theta = 1$ $\cos^2\theta = 1 - \sin^2\theta$ $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$ $\because \sin\theta = \frac{7}{12}$ $\cos = \pm\sqrt{1 - \left(\frac{7}{12}\right)^2}$ $\cos = \pm\sqrt{\frac{144-49}{144}}$ $\cos\theta = \pm\sqrt{\frac{95}{144}}$ $\boxed{\cos\theta = \frac{\sqrt{95}}{12}}$

	ii. $\cos 2\theta$. $\because \cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = \left(\frac{\sqrt{95}}{12}\right)^2 - \left(\frac{7}{12}\right)^2$ $\cos 2\theta = \frac{95}{144} - \frac{49}{144}$ $\cos 2\theta = \frac{95-49}{144}$ $\cos 2\theta = \frac{\cancel{46}\ 23}{\cancel{144}\ 72}$ $\boxed{\cos 2\theta = 23}$ iii. $\tan\theta$. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\tan\theta = \frac{\frac{7}{\cancel{12}}}{\frac{\sqrt{95}}{\cancel{12}}}$ $\boxed{\tan\theta = \frac{7}{\sqrt{95}}}$
Description of Weaker Responses	Weaker responses showcased a troubling pattern where candidates, despite correctly identifying the appropriate identity, struggle to execute the correct substitution. This reflected a gap in their understanding of how to apply these identities effectively to solve trigonometric problems.

Image of Weaker Response	 cos θ is opposite of sin θ $\frac{12}{7}$ is cos θ ii. Cos θ is multiplied by its original value and we multiply by 2 $2\left\lvert\frac{12}{7}\right\rvert = \frac{24}{14} = \frac{12}{7}$ $\frac{12}{7}$ is the answer for cos θ iii. $= \frac{\cos\theta}{\sin\theta}$ is known as tan θ $= \frac{7/12}{12/7}$ we get a new number = then we need to take square of both sides $\left(\frac{12}{7}\right)^2 = \left(\frac{7}{12}\right)^2 = \frac{144}{49} = \frac{49}{144}$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating)	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

• Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources		

Any Additional Suggestion:
Following are some approaches to help students grasp this concept.
Begin by ensuring that students have a solid understanding of the fundamental trigonometric ratios: sine, cosine, and tangent.
Explain that the given trigonometric ratio can be treated as a fraction, where one side of the ratio represents the length of one side of the triangle, and the other side represents the length of another side. This forms the basis for finding the remaining ratios.
Introduce trigonometric tables (sine, cosine, tangent tables) or calculator functions. Show students how to use these resources to find the values of remaining ratios.
Share incorrect solutions to trigonometric ratio problems and ask students to identify and correct errors. This helps them develop a critical eye and improve problem-solving skills.

Question No. 8i

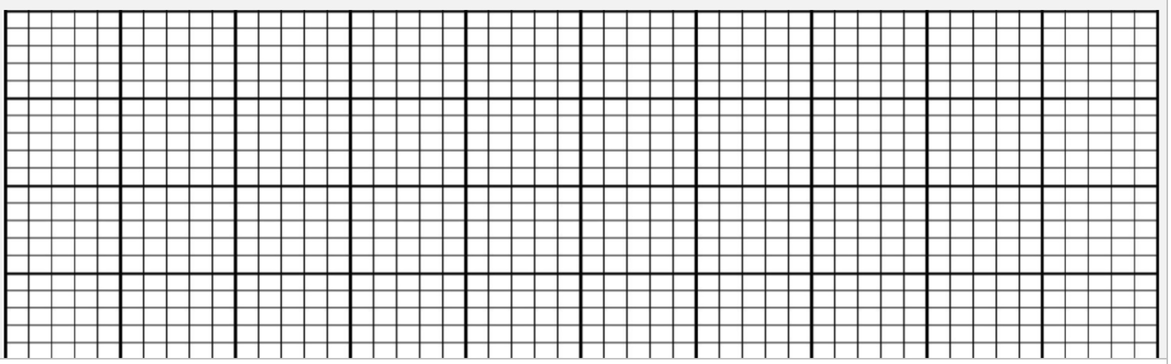
Question Text

A function is given as $y = \sin\frac{x}{2}$, where $-\pi \le x \le \pi$.

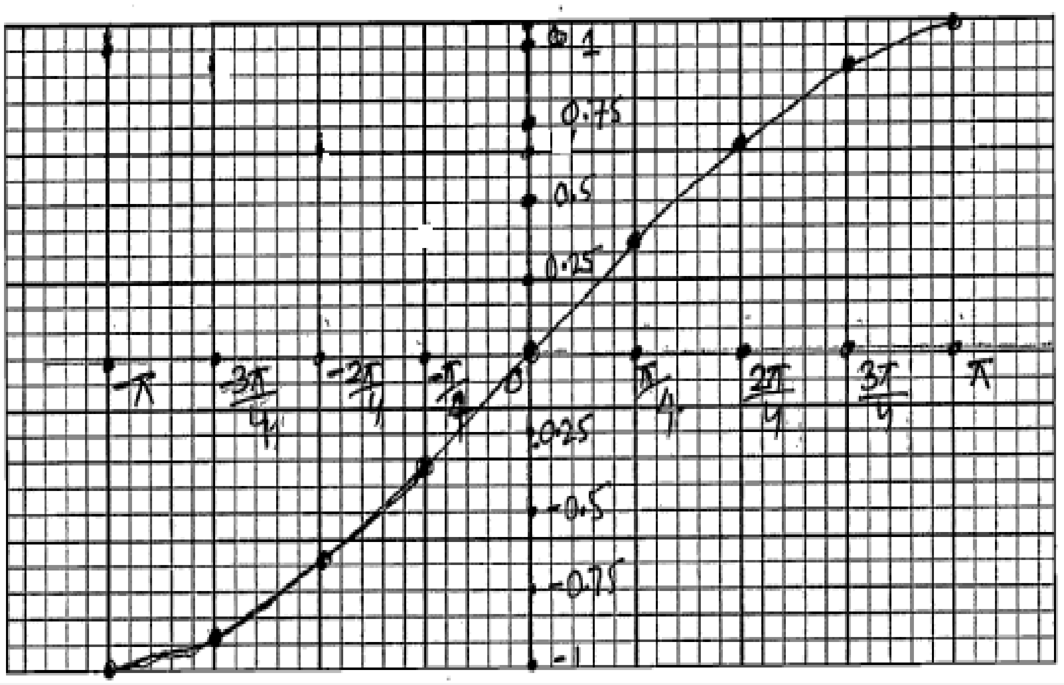
I. Complete the given table for the function.

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{4}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	π
$\sin\frac{x}{2}$									

Draw the graph of the function.



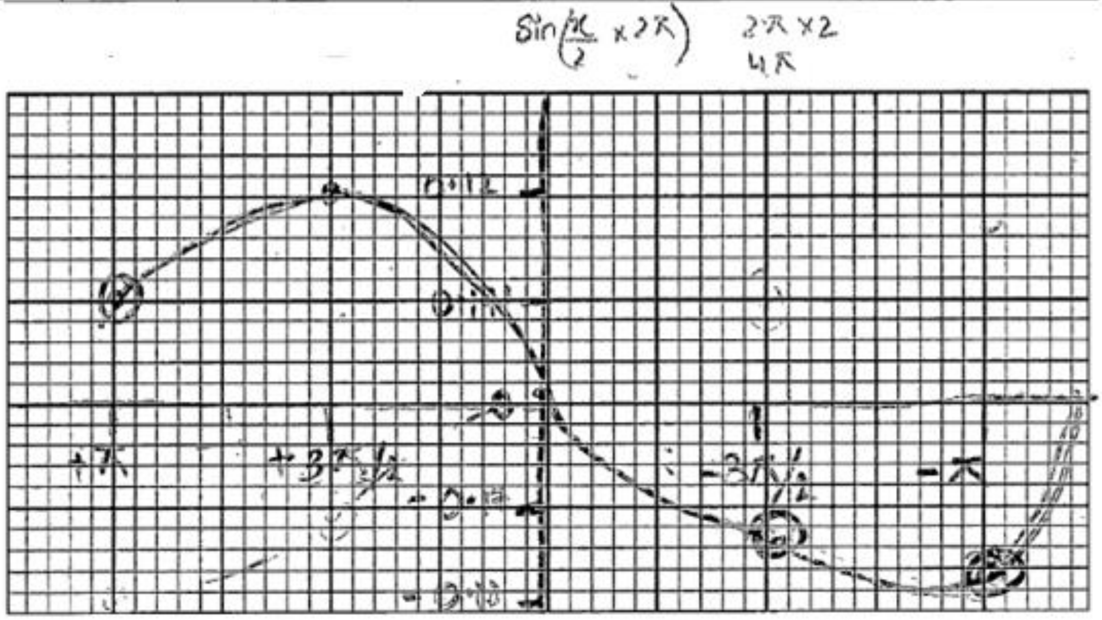
SLO No.	10.2.4
SLO Text	Sketch the trigonometric functions of the form $a\sin b\theta$, $a\cos b\theta$ and $a\tan b\theta$, where a and b are integers.
Max Marks	4
Cognitive Level	A

<table>
<tr><td>Checking Hints</td><td>i. 2 marks for completing the table (1 mark for each at least 4 values).
ii. 1 mark for drawing the scale on the given paper.
1 mark for plotting the points and connecting to draw the graph.</td></tr>
<tr><td>Overall Performance</td><td>This question consisted of two parts. Candidates were directed to attempt both. Part (i) was required to complete the table by calculating the corresponding values of x and hence, plot the ordered pairs in the graph paper. Overall, this question was solved correctly by few of the candidates only. However, many of the candidates made mistakes in calculating the values whereas there were some errors in the graph plotting.</td></tr>
<tr><td>Description of Better Responses</td><td>Better responses candidates completed the given tabulation with correct values of $\sin \frac{x}{2}$ against the given angles. they then precisely plotted these points on the given graph, ensuring an accurate representation of the given function.</td></tr>
<tr><td>Image of Better Response</td><td>
<table>
<tr><td>x</td><td>$-\pi$</td><td>$-\frac{3\pi}{4}$</td><td>$-\frac{2\pi}{4}$</td><td>$-\frac{\pi}{4}$</td><td>0</td><td>$\frac{\pi}{4}$</td><td>$\frac{2\pi}{4}$</td><td>$\frac{3\pi}{4}$</td><td>π</td></tr>
<tr><td>$\sin \frac{x}{2}$</td><td>-1</td><td>-0.92</td><td>-0.707</td><td>-0.38</td><td>0</td><td>0.38</td><td>0.707</td><td>0.92</td><td>1</td></tr>
</table>


</td></tr>
<tr><td>Description of Weaker Responses</td><td>Weaker responses, substituted incorrect values of x. Additionally, the errors in graphs included inappropriate scale and plotting of incorrect ordered pairs on the graph paper.</td></tr>
</table>

Image of Weaker Response

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{4}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	π
$\sin\frac{x}{2}$	-0·171	-0·12	-0·086	-0·043	0	0·043	0·086	0·12	0·171



Suggestions for Improvement (Highlighted part)

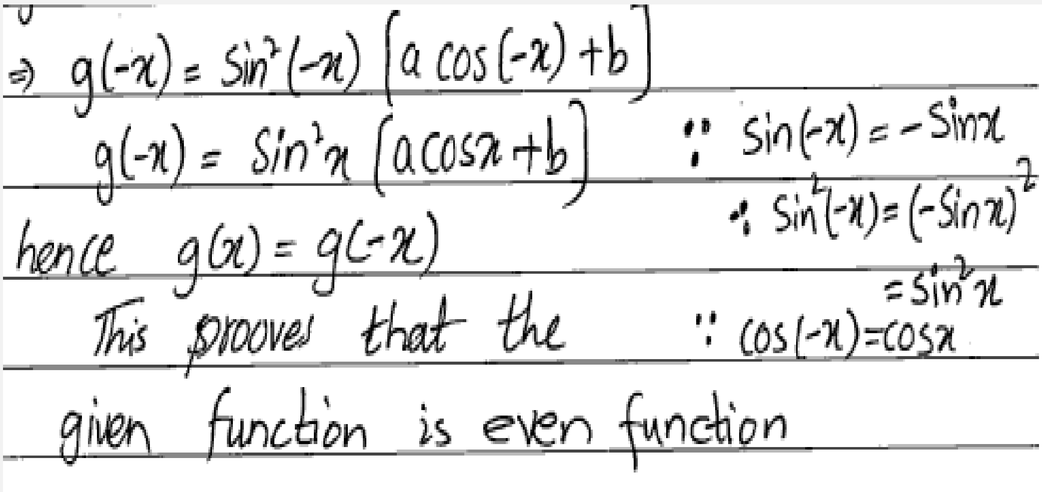
How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Knowledge Platform real Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

Following are some teaching strategies related to misconceptions and common errors which candidates exhibited.

Provide tutorials on how to create accurate graphs, emphasising the importance of selecting an appropriate scale and labelling axes correctly.

Use graphing software or online tools that generate graphs with adjustable scales. This allows students to experiment with different scales and see their impact.
Graph Interpretation: Analyse existing graphs and discuss their features, including scale, labels, and plotted points. Help students understand the significance of each component.
Provide graphing templates with pre-labelled axes and scales for practice. This can help students focus on plotting data accurately.
Provide datasets for students to analyse and interpret. Encourage them to draw conclusions and insights from the data they've tabulated and graphed.
Step-by-Step Guided Practice: Start with guided exercises that demonstrate the correct process of substituting values into equations or functions. Walk students through each step.

Question No. 8ii	
Question Text	Prove that the function $g(x)=\sin^2 x(a\cos x+b)$ is an even function.
SLO No.	10.1.2
SLO Text	Distinguish between even and odd trigonometric functions.
Max Marks	2
Cognitive Level	U
Checking Hints	1 mark for writing $\sin^2 x(a\cos x+b)$ as $[\sin(-x)]^2[a\cos(-x)+b]$ 1 mark for writing $\sin(-x)=-\sin x$ and $\cos(-x)=\cos x$ 1 mark for writing $g(-x)=(\sin x)^2(a\cos x+b)$ to get the required proof.
Overall Performance	Question 8(ii) was related to concept of even function. Majority of the candidates did not attempt this question correctly or did not attempt it.
Description of Better Responses	Better responses replaced x with $-x$ and applied the properties of sine and cosine functions. Such responses, were able to prove that the given function was an even function.
Image of Better Response	 $\Rightarrow g(-x) = \sin^2(-x)[a\cos(-x)+b]$ $g(-x) = \sin^2 x[a\cos x+b]$ $\because \sin(-x) = -\sin x$ $\therefore \sin^2(-x) = (-\sin x)^2 = \sin^2 x$ hence $g(x) = g(-x)$ $\because \cos(-x) = \cos x$ This prooves that the given function is even function
Description of Weaker Responses	In weaker responses, the candidate's concept of even functions was not clear. Some of the candidates applied identities and used incorrect simplifications to prove. While others equated $a\cos x+b$ with $\sin^2 x$ to prove the given function.

Images of Weaker Responses

Image (i)

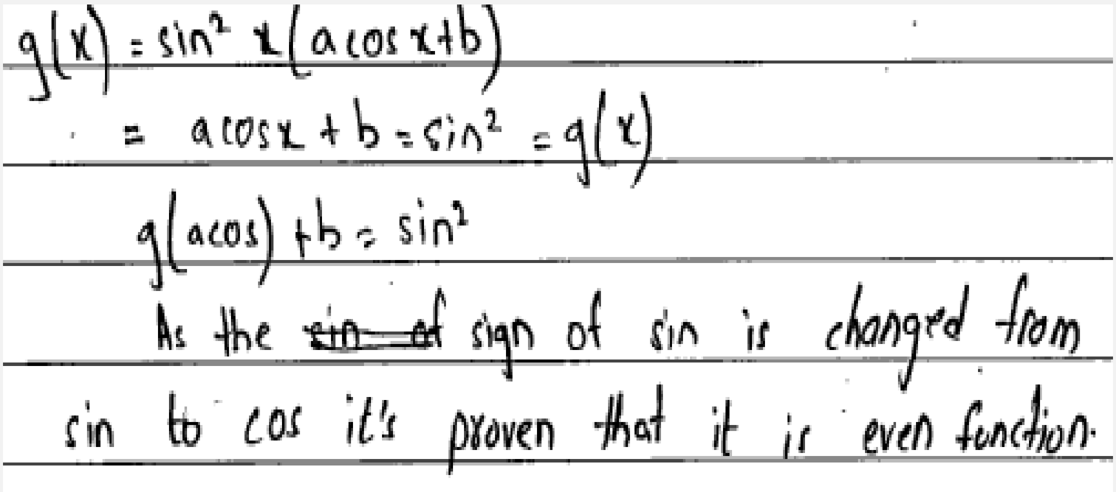
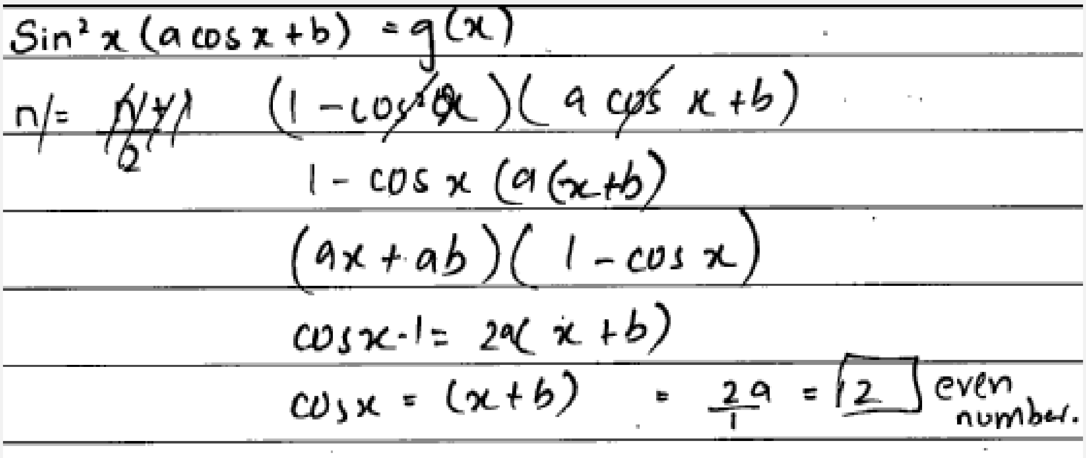

$g(x) = \sin^2 x(a\cos x + b)$
$= a\cos x + b = \sin^2 = g(x)$
$g(a\cos) + b = \sin^2$
As the ~~sin of~~ sign of sin is changed from sin to cos it's proven that it is even function.

Image (ii)


$\sin^2 x(a\cos x + b) = g(x)$
$n/= \cancel{f(x)/2}$ $(1 - \cancel{\cos^2 x})(a\cancel{\cos}\, x + b)$
$1 - \cos x\,(a(x+b))$
$(ax + ab)(1 - \cos x)$
$\cos x - 1 = 2a(x+b)$
$\cos x = (x+b) = \frac{2a}{1} = \boxed{2}$ even number.

Suggestions for Improvement (Highlighted part)

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• Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources		

Any Additional Suggestion:
Following are some activities and strategies to effectively teach the concept.
Start by plotting the graph of the given function. Emphasise that even functions have symmetry about the y-axis. Discuss how the graph looks the same on both sides of the y-axis.
Let students select various x-values and calculate $f(x)$ and $f(-x)$ for the given function. Encourage them to use different values to see if the equality $f(x) = f(-x)$ holds true.
Introduce the concept of symmetry and mirror images. Explain that even functions exhibit reflectional symmetry, which means they look the same when reflected across the y-axis.
Teach students how to prove a function's evenness algebraically. Start with a generic function, f(x), and show that replacing x with $-x$ results in $f(-x)$. Simplify the equation and conclude that $f(x) = f(-x)$.
Walk students through step-by-step examples of proving a function's evenness. Start with simple functions and gradually move to more complex ones.
Share incorrect proofs or arguments and ask students to identify the errors. This helps them develop critical thinking skills and recognise common mistakes.
Discuss real-life situations where even functions are encountered, such as in physics (reflection and symmetry) or engineering (signal processing). Show how even functions are used in these contexts.

Annexure A: Pedagogies Used for Teaching the SLOs

Pedagogy: Storyboard
Description: A visual pedagogy that uses a series of illustrated panels to present a narrative, encouraging creativity and critical thinking. It helps learners organise ideas, sequence events, and comprehend complex concepts through storytelling.
Example: In a Literature class, students are tasked with creating storyboards to visually retell a novel. They draw key scenes, write captions, and present their stories to the class, enhancing their reading comprehension and fostering their imagination.

Pedagogy: Cause and Effect
Description: This pedagogy explores the relationships between actions and consequences. By analysing cause-and-effect relationships, learners develop a deeper understanding of how events are interconnected and how one action can lead to various outcomes.
Example: In a History class, students study the causes and effects of the Industrial Revolution. They research and discuss how technological advancements in manufacturing led to significant societal changes, such as urbanisation and labour reform movements.

Pedagogy: Fish and Bone
Description: A method that breaks down complex topics into main ideas (the fish) and supporting details (the bones). This visual approach enhances comprehension by highlighting essential concepts and their relevant explanations.
Example: During a Biology class on human anatomy, the teacher uses the fish and bone technique to teach about the human skeletal system. Teacher presents the main components of the human skeleton (fish) and elaborates on each bone's structure and function (bones).

Pedagogy: Concept Mapping
Description: An effective way to visually represent relationships between ideas. Learners create diagrams connecting key concepts, aiding in understanding the overall structure of a subject and fostering retention.
Example: In a Psychology assignment, students use concept mapping to explore the various theories of personality. They interlink different theories, such as Freud's psychoanalysis, Jung's analytical psychology, and Bandura's social-cognitive theory, to see how they relate to each other.

Pedagogy: Audio Visual Resources
Description: Incorporating multimedia elements like videos, images, and audio into lessons. This approach caters to different learning styles, making educational content more engaging and memorable.
Example: In a General Science class, the teacher uses a documentary-style video to teach about the solar system. The video includes stunning visual animations of the planets, interviews with astronomers, and background music, enhancing students' interest and understanding of space.

Pedagogy: Think, Pair, and Share
Description: A collaborative learning technique where students ponder a question or problem individually, then discuss their thoughts in pairs or small groups before sharing with the entire class. It fosters active participation, communication skills, and diverse perspectives.
Example: In a Literature in English class, the teacher poses a thought-provoking question about a novel's moral dilemma. Students first reflect individually, then pair up to exchange

their opinions, and finally participate in a lively class discussion to explore different viewpoints.

Pedagogy: Questioning Technique (Socratic Approach)
Description: Based on Socratic dialogue, this method stimulates critical thinking by posing thought-provoking questions. It encourages learners to explore ideas, justify their reasoning, and discover knowledge through a process of inquiry.
Example: In an Ethics class, the instructor uses the Socratic approach to lead a discussion on the meaning of justice. By asking a series of probing questions, the students engage in a deeper exploration of ethical principles and societal values.

Pedagogy: Practical Demonstration
Description: A hands-on approach where learners observe real-life applications of theories or skills. Practical demonstrations enhance comprehension, skill acquisition, and problem-solving abilities by bridging theoretical concepts with real-world scenarios.
Example: In a Food and Nutrition class, the instructor demonstrates the proper technique for filleting a fish. Students observe and then practice the skill themselves, learning the practical application of knife skills and culinary precision.

(**Note:** The examples provided in this annexure serve as illustrations of various pedagogies. It is important to understand that these pedagogies are versatile and can be applied across subjects in numerous ways. Feel free to adapt and explore these techniques creatively to enhance learning outcomes in your specific context.)

Acknowledgements

The Aga Khan University Examination Board (AKU-EB) acknowledges with gratitude the invaluable contributions of all the dedicated individuals who have played a pivotal role in the development of the Mathematics HSSC-I E-Marking Notes.

We extend our sincere appreciation to Ms Shahneel Jawaid and Mr Mukhtar Ali Khan, Specialist and former Specialist in Mathematics at AKU-EB, for taking subject lead during the entire process of e-marking.

We particularly thank to Mr Faizan Ahmed, Senior Marker, Aga Khan Higher Secondary School Karimabad, Karachi, for evaluating each question's performances, delineating strengths and weaknesses in candidates' responses, and highlighting instructional approaches along with recommendations for better performance.

Additionally, we express our gratitude to the esteemed team of reviewers for their constructive feedback on overall performance, better and weaker responses, and validating teaching pedagogies along with suggestions for improvement.

These contributors include:

- Dr Sumera Anjum, Lead Specialist, Curriculum Development, AKU-EB
- Rabia Nisar, Specialist in Assessment, AKU-EB
- Munira Muhammad, Lead Specialist, Assessment, AKU-EB
- Zain Muluk, Manager, Examination Development, AKU-EB
- Raabia Hirani, Manager, Curriculum Development, AKU-EB
- Ali Aslam Bijani, Manager, Teacher Support, AKU-EB
- Dr Shehzad Jeeva, CEO, AKU-EB