AGA KHAN UNIVERSITY EXAMINATION BOARD

Notes from E-Marking Centre HSSC-II Mathematics Annual Examinations 2023

Introduction

This document has been prepared for the teachers and candidates of Higher Secondary School Certificate (HSSC) Part II (Class XII) Mathematics. It contains comments on candidates' responses to the 2023 HSSC-II Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that requires candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations

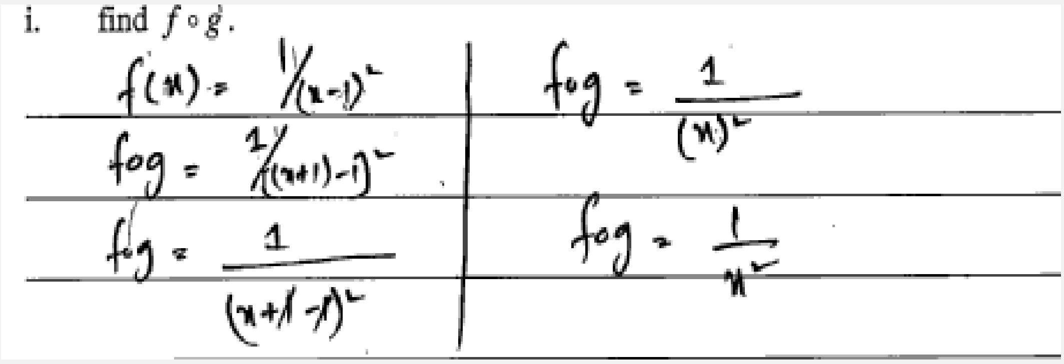
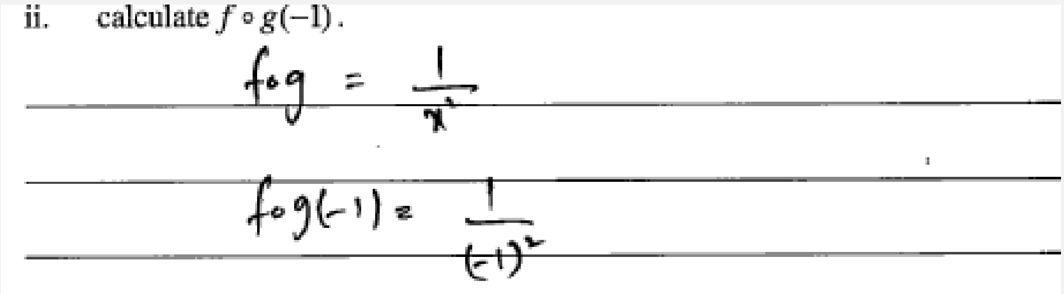
Candidates performed really well in some concepts, such as, Chain Rule, Composite Functions and Analytic Geometry. However, candidates who did not score well mostly failed to understand the demands of the questions, often misinterpreting the command words and the stimuli.

Mentioned below are a few concepts that teachers need to focus so that the candidates may perform better.

- Derivative of Trigonometric Functions
- Techniques of Integration
- Plotting of linear Inequality, Feasible Region and Corner Points
- Circle and Tangent of the Circle
- Vector Algebra

Note: Candidates' responses shown in this report have not been corrected for grammar, spelling, format, or information.

DETAILED COMMENTS
Constructed Response Questions (CRQs)

Question No. 1	
Question Text	It is given that $f(x)=\frac{1}{(x-1)^2}$, $x\neq 1$ and $g(x)=x+1$ are two real valued functions. For the given functions, i. find $f\circ g$. ii. calculate $f\circ g(-1)$. iii. find $g^{-1}(x)$.
SLO No.	12.2.2/12.4.2/12.5.2
SLO Text	Find the composition of two functions. Find the corresponding values of composite functions for given values of a variable. Find the inverse of a function and its domain and range.
Max Marks	3
Cognitive Level	A*
Checking Hints	i. 1 mark for substituting $g(x)=x+1$ for x in $f(x)$ ii. 1 mark for substituting for $x=-1$ in the $f\circ g$ iii. 1 mark for substituting $y=g(x)$ and rearrange to get $g^{-1}(x)$
Overall Performance	Most of candidates successfully tackled this question, which involved composite functions, finding the value of a function at specific points, and understanding inverse functions. Such candidates followed all the required steps and met the demands of the question. However, few of the candidates faced some challenges with substituting the correct value of g in $f\circ g$.
Description of Better Responses	The three parts of the question were related to composite function, value of function at any specified point and the inverse function. The better responses demonstrated grasp on these concepts through detailed and step by step application of the rules to find the mentioned.
Image of Better Response	 i. find $f\circ g$. $f(x) = 1/(x-1)^2$ $fog = 1/((x+1)-1)^2$ $fog = \frac{1}{(x+1-1)^2}$ $fog = \frac{1}{(x)^2}$ $fog = \frac{1}{x^2}$  ii. calculate $f\circ g(-1)$. $fog = \frac{1}{x^2}$ $fog(-1) = \frac{1}{(-1)^2}$

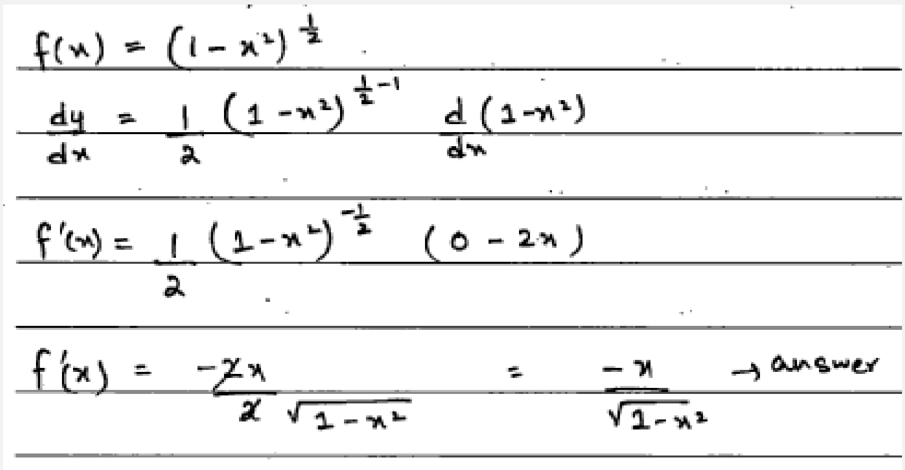
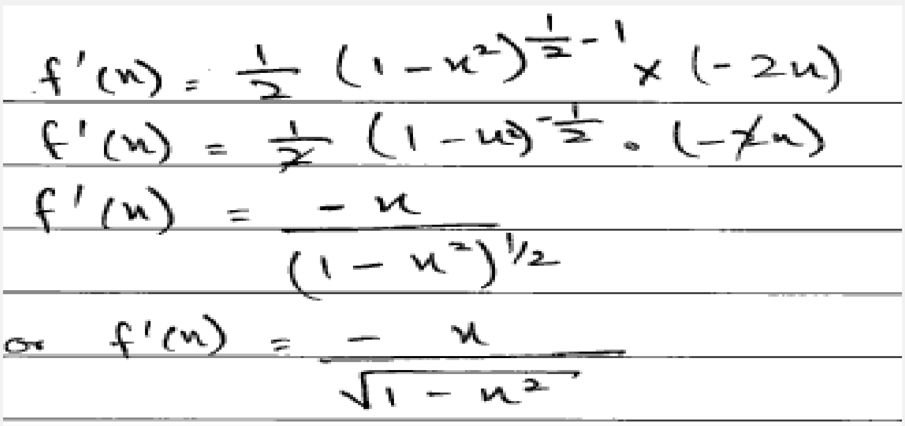
	iii. find $g^{-1}(x)$. $g(x) = x+1$ $y = x+1$ $y-1 = x$ $g^{-1}(x) = x-1$ $g^{-1}(x) = (x-1)$
Description of Weaker Responses	Weaker responses showed inappropriate substitutions (in case of composite functions) and errors in simplification. Other observed errors included multiplication of the given functions or equating the functions with each other. Few errors were also observed in finding the value of the function. In inverse function, some candidates took the reciprocal the function.
Image of Weaker Response	$\frac{1}{(x-1)^2} + \frac{x+1}{1} = \frac{1+x+1}{(x-1)^2(1)}$ $\cancel{1 + (x+1}$ $= \frac{2x}{(x-1)^2}$ ii. calculate $f \circ g(-1)$. $\frac{1}{(x-1)^2} = x+1$ $\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$ $f(-1)\, g'(-1) dx = f(-1)\, g(-1) - \int f'(-1)\, g(-1)\, dx$ iii. find $g^{-1}(x)$. $g^{-1} = (x+1)$ $g^{-1} = (x-1)$ $g(x-1) = (x+1)$ $g = \frac{x+1}{x-1}$

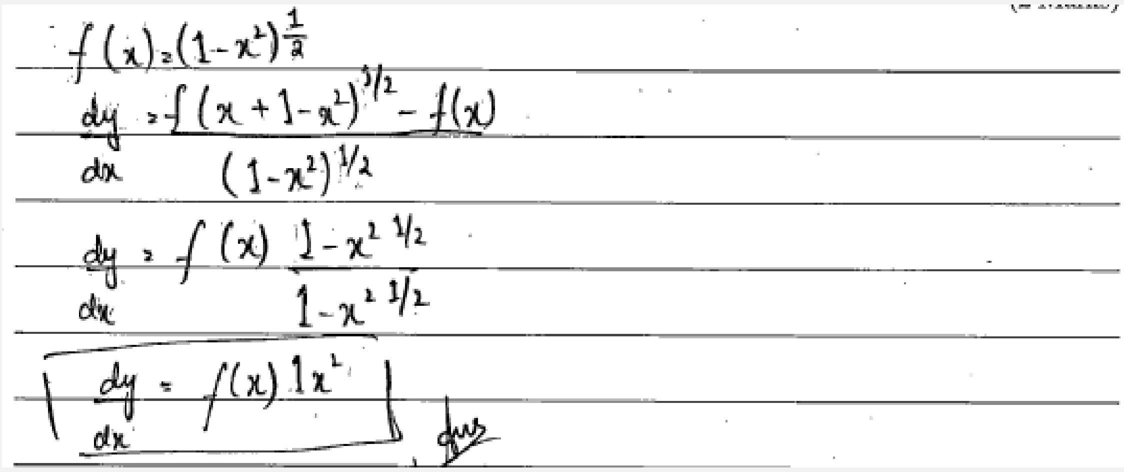
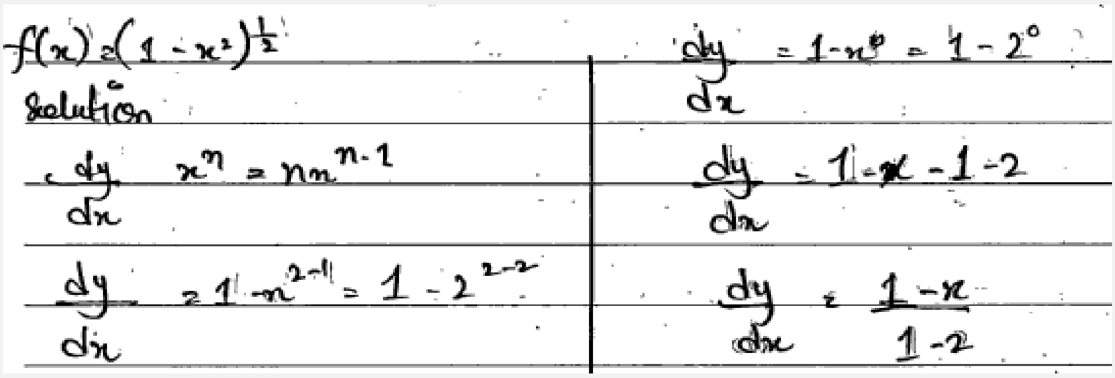
Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy** Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration ** For description of each pedagogy, refer to Annexure A	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
To enhance better understanding of the concept, teachers are recommended to use the following strategies.
Visual Representation: Use graphs or diagrams to visually illustrate function composition. Show how the output of one function becomes the input of another. Graph both the original function and its inverse to visually demonstrate their relationship. Discuss how the graphs are reflections across the line $y = x$.
Interactive Activities: Incorporate interactive tools or software that allow students to input functions and see the composition results in real-time.
Real-Life Application: Showcase how function composition is applied in fields like economics, physics, or engineering. Connect the concept to practical scenarios. Connect function inverses to real-life scenarios where "reversing" an operation is essential, such as interest rate calculations or growth models.
Guided Practice: Give students practice problems with different functions to compose. Walk them through each step, highlighting the importance of proper order and input substitution.

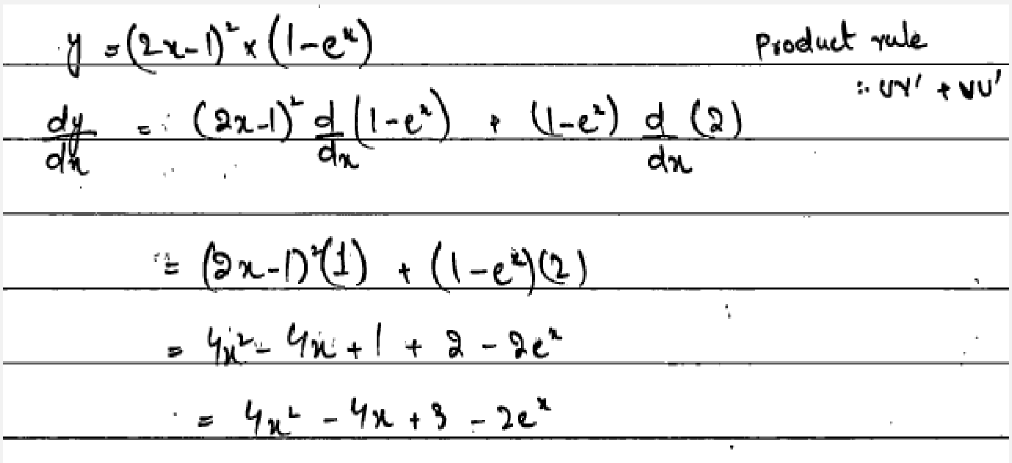
*K = Knowledge   U = Understanding   A = Application and other higher-order cognitive skills

Question No. 2ai Candidates were given the choice to attempt any TWO out of the three questions: 2a, 2b and 2c.	
Question Text	Find the derivative of the function $f(x) = \left(1 - x^2\right)^{\frac{1}{2}}$ with respect to x.
SLO No.	13.1.8
SLO Text	Find the derivative of algebraic functions by using direct method (power rule).
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for differentiating the external power of the expression. 1 mark for differentiating the inner function i.e. $\left(1 - x^2\right)$.
Overall Performance	Majority of candidates attempted question 2ai and demonstrated an ability to apply the power rule for differentiating the provided function. This reflects a clear understanding of fundamental differentiation techniques. However, a subset of candidates encountered challenges when employing the power rule, indicating the need for targeted support in mastering this concept.
Description of Better Responses	Better responses showed the correct application of the direct rule of derivatives and completing the differentiation process without any error in mathematical operations.
Images of Better Responses	**Image (i)**  $f(x) = (1-x^2)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{\frac{1}{2}-1} \frac{d}{dx}(1-x^2)$ $f'(x) = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(0-2x)$ $f'(x) = \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$ → answer **Image (ii)**  $f'(x) = \frac{1}{2}(1-x^2)^{\frac{1}{2}-1} \times (-2x)$ $f'(x) = \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x)$ $f'(x) = \frac{-x}{(1-x^2)^{1/2}}$ or $f'(x) = \frac{-x}{\sqrt{1-x^2}}$

Description of Weaker Responses	Weaker responses revealed that candidates could not recall and apply the power rule of derivation. Some of the candidates applied the power rule, but did not apply the derivative on the inner function (that was base function). Moreover, some candidates either differentiated the terms separately or differentiated only the power of the function.
Images of weaker Responses	**Image (i)**  **Image(ii)** 
Question No. 2aii	
Question Text	For $y=(2x-1)^2\times(1-e^x)$, find $\frac{dy}{dx}$.
SLO No.	13.2.1
SLO Text	Find the derivative of d) product of two functions.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for writing the function in terms of product rule. 1 mark for differentiating $1-e^x$ correctly. 1 mark for differentiating $(2x-1)^2$ correctly.
Overall Performance	Majority of candidates attempted this question with accurate results, indicating a satisfactory understanding of the concept. While there were few candidates who did not meet the question requirement and made mistakes in applying product rule correctly.
Description of Better Responses	Better responses showed the correct solution process by correctly applying the product rule and correct differentiation of the functions separately.

Images of Better Responses	**Image (i)** $\frac{dy}{dx} = (2x-1)^2 \cdot (-e^x) + (1-e^x)\, 2(2x-1)(2)$ $= (2x-1)^2 \cdot (-e^x) + (1-e^x)\, 4(2x-1)$ $= (4x^2+4x+1)(-e^x) + (1-e^x)(8x-4)$ $= -4e^x x^2 - 4e^x x - e^x + 8x - 4 - 8e^x x + 4e^x$ $\frac{dy}{dx} = -4e^x x^2 - 12e^x x + 3e^x + 8x - 4$ **Image (ii)** $\frac{dy}{dx} = \frac{d}{dx}(2x-1)^2 \cdot (1-e^x)$ $\frac{d}{dx}(f(x) \cdot g(x)) = fx \frac{d}{dx} gx + gx \frac{d}{dx} fx$ $(2x-1)^2 \frac{d}{dx}(1-e^x) + (1-e^x) \frac{d}{dx}(2x-1)^2$ $= (2x-1)^2 \cdot -e^x + (1-e^x) \cdot 2(2x-1) \cdot 2$ $= -(2x-1)^2 \cdot e^x + 4(1-e^x)(2x-1)$ $= \boxed{(2x-1)\left[-(2x-1) + 4(1-e^x)\right]}$
Description of Weaker Responses	Weaker responses exhibited that there were challenges in correct application of the product rule, even though this formula was provided in the formula sheet. Moreover, there were errors in finding the derivatives of the separate functions during applying the product rule. Specifically, some of the candidates made errors in the derivative of the exponential function.
Images of weaker Responses	**Image (i)** $y = (2x-1)^2 \times (1-e^x)$ $u = (2x-1)^2 = u' = 2(2x-1)$ $v = (1-e^x) = v = (1-e^x)$ $= (2x-1)^2(1\cancel{e}-e^x) + (1-e^x)2(2x-1)$ $= 2x^2 - 2(2x) - 1^2 (1-e^x) + (4x-2)(1-e^x)$ $= \cancel{4x} = 2(2x-1)(1-e^x)$ Ans.

Image (ii)



$$y = (2x-1)^2 \times (1-e^x)$$ Product rule $= uv' + vu'$

$$\frac{dy}{dx} = (2x-1)^2 \frac{d}{dx}(1-e^x) + (1-e^x)\frac{d}{dx}(2)$$

$$= (2x-1)^2(1) + (1-e^x)(2)$$

$$= 4x^2 - 4x + 1 + 2 - 2e^x$$

$$= 4x^2 - 4x + 3 - 2e^x$$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

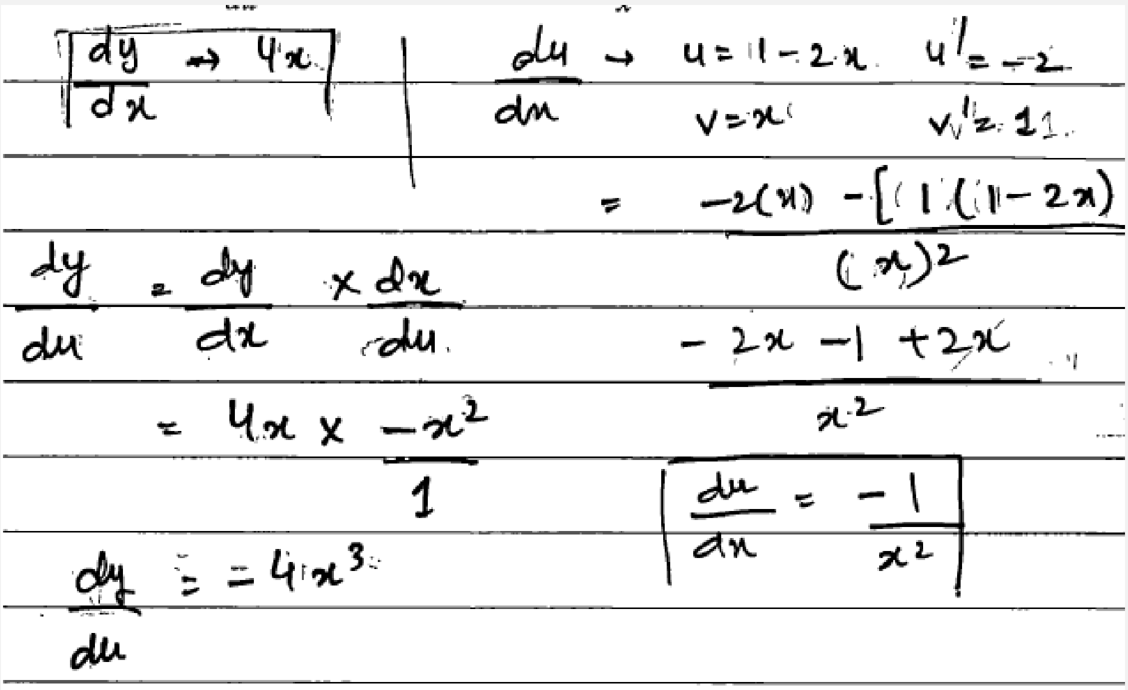
While teaching the concept of derivatives, teachers are advised to use the following strategies. These strategies collectively contribute to improving accuracy and confidence in differentiation outcomes.

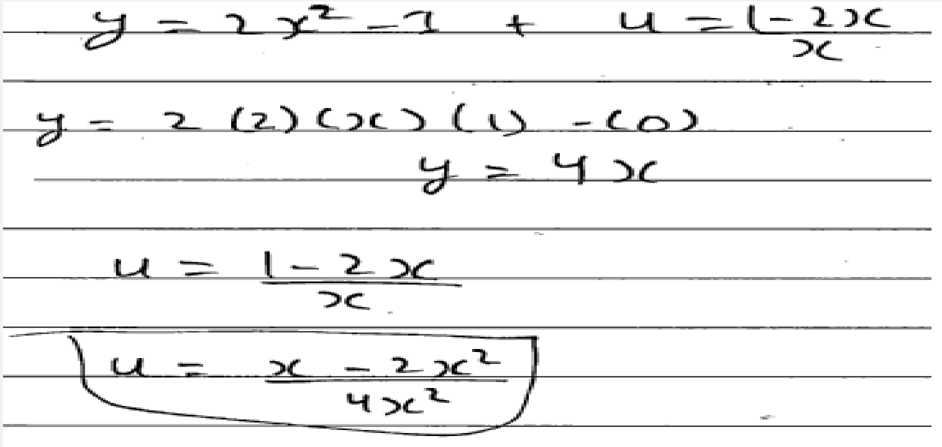
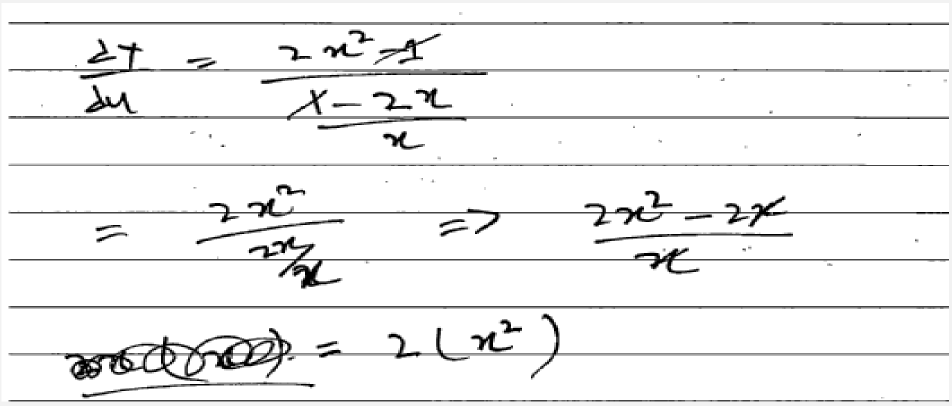
Offer focused practice exercises emphasising step-by-step product rule application.

Break down problems into smaller components to aid comprehension of derivative calculations for each function.

Address challenges related to the exponential function's derivatives with different algebraic expressions examples and explanations.

Advise students to thoroughly review the formulae included in the provided formula sheet.

Question No. 2bi Candidates were given the choice to attempt any TWO out of the three questions: 2a, 2b and 2c.	
Question Text	Find $\frac{dy}{du}$ for $y = 2x^2 - 1$ and $u = \frac{1-2x}{x}$.
SLO No.	13.3.2
SLO Text	Solve problems related to chain rule.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for finding $\frac{dy}{dx} = 4x$. 1 mark for finding $\frac{du}{dx} = \frac{-1}{x^2}$. 1 mark for correctly feeding the derivatives in the chain rule.
Overall Performance	Relatively lesser number of candidates attempted 2b. This part of 2b, i.e., 2bi tested candidates' ability to differentiate one function with respect to another function employing the chain rule of differentiation. Overall, majority of the candidates attempting this question scored full marks. However, there were few candidates who attempted this question partially or did not attempt at all.
Description of Better Responses	Better response showed candidates' ability to apply the chain rule (provided in the formula sheet). In addition to this, the candidates also found the derivatives involved in the chain rule correctly.
Image of Better Response	 $\boxed{\frac{dy}{dx} \rightarrow 4x}$ $\frac{du}{dx} \rightarrow u = 1-2x \quad u' = -2$ $v = x \quad v' = 1$ $= \frac{-2(x) - [1(1-2x)]}{(x)^2}$ $\frac{-2x - 1 + 2x}{x^2}$ $\boxed{\frac{du}{dx} = \frac{-1}{x^2}}$ $\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du}$ $= 4x \times \frac{-x^2}{1}$ $\frac{dy}{du} = 4x^3$
Description of Weaker Responses	Weaker responses reflected the gap in understanding of the chain rule and its application. Many of the candidates applied quotient rule, which was not the appropriate or correct method for differentiation of such functions.

Images of weaker Responses	**Image (i)**  **Image (ii)** 
Question No. 2bii	
Question Text	For $y = \sin\sqrt{2x}$, find $\frac{dy}{dx}$.
SLO No.	13.4.2
SLO Text	Find the derivative of trigonometric functions using direct method.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for writing $\cos\sqrt{2x}$ i.e., differentiating the external function. 1 mark for finding derivative of the angle and writing in the form $\cos\sqrt{2x}\times\frac{1}{2}\times\frac{1}{\sqrt{2x}}\times 2$.
Overall Performance	This part of 2b, i.e., 2bii tested candidates' ability to differentiate a trigonometric function. Overall, only few of the candidates attempting this question scored full marks. However, there were candidates who attempted this question partially or did not attempt at all.
Description of Better Responses	Better responses showed the correct differentiation of the external function (sine) and further applied the derivative to the angle function. Thus, better responses showed the correct differentiation of both these functions as per requirement.

<table>
<tr>
<td>Images of Better Responses</td>
<td>Image (i)
$\frac{dy}{dx} = \frac{d}{dx}(\sin\sqrt{2x})$
$\frac{dy}{dx} = \cos\sqrt{2x}\ \frac{d}{dx}\{(2x)^{1/2}\}$
$\frac{dy}{dx} = \cos\sqrt{2x}\ \left(\frac{1}{2}\ (2x)^{-1/2}\ (2)\right)$
$\frac{dy}{dx} = \cos\sqrt{2x}\left(\frac{2}{2\sqrt{2x}}\right)$
$\frac{dy}{dx} = \frac{2}{2\sqrt{2x}}\ \cos\sqrt{2x}$ ans.

Image (ii)
$y = \sin\sqrt{2x}$
Diff with respect to x
$\frac{dy}{dx} = \frac{d}{dx}\ \sin\sqrt{2x}$
$\frac{dy}{dx} = \cos\sqrt{2x} \cdot \frac{d}{dx}\sqrt{2x}$
$\frac{dy}{dx} = \cos\sqrt{2x} \cdot \frac{1}{2}\ (2x)^{-1/2} \cdot \frac{d}{dx}\ (2x)$
$\frac{dy}{dx} = \cos\sqrt{2x} \cdot \frac{1}{\cancel{2}\sqrt{2x}} \cdot \cancel{2}$
$\boxed{\frac{dy}{dx} = \frac{\cos\sqrt{2x}}{\sqrt{2x}}}$</td>
</tr>
<tr>
<td>Description of Weaker Responses</td>
<td>Weaker responses showed candidates’ inability to differentiate trigonometric function correctly. Hence, they made mistakes in applying the trigonometric differentiation, did not differentiate the angle or made both the mistakes. Also, some candidates put the power of the angle to the power of the trigonometric function or applied the product rule between the trigonometric sine function and its angle or could not differentiate the angle separately.</td>
</tr>
<tr>
<td>Images of weaker Responses</td>
<td>Image (i)
$y = \sin(2x)^{1/2}$
$\frac{dy}{dx} = \frac{d}{dx}(\sin(2x)^{1/2})$
$\frac{dy}{dx} = \frac{1}{2}(2\sin x \cos x)$
$\frac{dy}{dx} = \sin x\ \frac{\cos x}{2}$</td>
</tr>
</table>

	Image (ii) $\frac{dy}{dx} = \sin \frac{d}{dx}\sqrt{2x} + \sqrt{2x}\,\frac{d}{dx}\sin$ $= \sin \cdot \frac{1}{2}\, 2x^{-1/2} + \sqrt{2x} \cdot \cos$ $= \sin \cdot \frac{1}{2\sqrt{2x}} + \sqrt{2x}\cos$ $= \frac{\sin}{2\sqrt{2x}} + \sqrt{\cos 2x}$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

While teaching these concepts, teachers are advised to use the following strategies. These strategies collectively contribute to improving accuracy and confidence in differentiation outcomes.

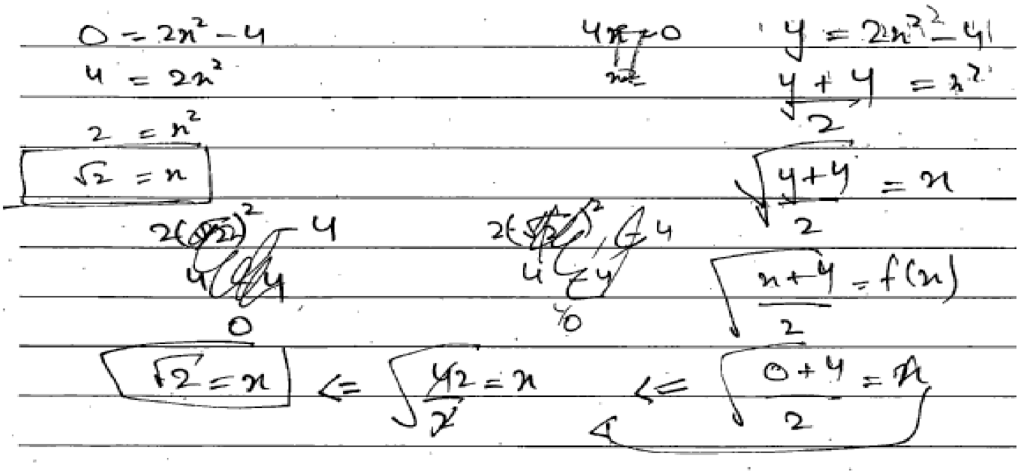
Step-by-Step Breakdown: Break down the chain rule into its components. Explain how to identify the inner function, the outer function, and their respective derivatives. Guide students through each step of the process with clear explanations.

Interactive Activities: Design interactive activities or online tools i.e., Desmos and wolframalpha that allow students to experiment with different functions and observe how the chain rule impacts the result. This hands-on approach can enhance their intuition about the rule.

Use of Technology: Utilise graphing calculators, mathematical software, or online platforms that can automatically compute derivatives using the chain rule. This can help students focus on the concept without getting bogged down by tedious calculations. Providing step-by-step examples that highlight the separation of the trigonometric function and its angle during differentiation can aid in understanding. Emphasising the importance of differentiating the angle separately and then applying the derivative to the trigonometric function is crucial.

Question No. 2ci Candidates were given the choice to attempt any TWO out of the three questions: 2a, 2b and 2c.	
Question Text	Find the equation of tangent to the curve $f(x) = x^2 - 4x + 5$ at $(1, -1)$.
SLO No.	14.3.2
SLO Text	Find the equation of tangent and normal to the curve at a given point.
Cognitive Level	A
Checking Hints	1 mark for finding first derivative 1 mark for value of the first derivative at the given point 1 mark for correct substitution in the point slope form
Overall Performance	Relatively lesser number of candidates, as compared to previous two parts, attempted 2c. This part of 2c, i.e., 2ci tested candidates' ability to apply derivative to find the slope of the tangent to a curve. Overall, a lesser number of the candidates attempting this question scored full marks. Most of the candidates solved this question partially correct.
Description of Better Responses	Candidates with better responses found the derivative of the function and substituted the given value of *x* to find the slope of the tangent at the specified point. Moreover, those candidates employed the point slope form to find the equation of the tangent.
Image of Better Response	Taking derivative of the function $\frac{dy}{dx} = 2x - 4$ putting value of $(1, -1)$ $\frac{dy}{dx} = 2(1) - 4 \Rightarrow \frac{dy}{dx} = -2$ So slope is -2 Using point slope form $y - (-1) = -2(x - 1)$ $y + 1 = -2x + 2$ $y + 2x = 1$ — equation of tangent
Description of Weaker Responses	Weaker responses showed candidates' conceptual gap in relating the slope of tangent to the derivative of a function. Moreover, candidates showed lack of understanding of the substituting the value of *x* in the derivative to find the slope of the tangent. In addition, some candidates used the direct formula for finding the tangent to a circle.

Image of weaker Response	Given Tangent to the curve $f(x)=x^2-4x+5$ At the points $=(1,-1) \longrightarrow (x_1, y_1)$ Required is equation at the tangent using formula: $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ Put points (x_1, y_1) $1x-1y+g(x+1)+f(y-1)+c=0$ $x-y+gx+g+fy-f+c=0$ Answer
Question No. 2cii	
Question Text	Find the *x*-coordinate of the point on the curve $f(x)=2x^2-4$ at which the tangent to $f(x)$ is parallel to the horizontal axis.
SLO No.	14.3.4
SLO Text	Find the point on a curve where the tangent is parallel to the given line.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for finding first derivative. 1 mark for putting $4x=0$ and evaluating *x*.
Overall Performance	This part of 2c, i.e. 2cii tested candidates' ability to apply the geometrical interpretation of derivative. Overall, a lesser number of the candidates attempting this question scored full marks. Most of the candidates solved this question partially correct.
Description of Better Responses	Better responses differentiated the given function and equated it to zero. Hence, evaluated correctly for *x*. Thus, they obtained the point on which the tangent to the given curve was parallel to the *x* – axis.
Image of Better Response	As we know that slope of horizontal axis is zero, the slope of tangent to $f(x)$ will also be zero. $y=2x^2-4$ $\frac{dy}{dx}=4x \rightarrow 0=4x$ $x=0$ $\boxed{x \text{ coordinate is } 0}$
Description of Weaker Responses	Candidates with weaker responses could not relate derivative of a function with the slope of the tangent. Moreover, they could not evaluate for *x* by equating the derivative of the given function to zero. Some candidates demonstrated some relationship of derivative with

	the tangent as they differentiated the function and equated to zero. However, due to incorrect differentiation, they could not evaluate the correct value of *x*.
Image of weaker Response	

Suggestions for Improvement (Highlighted part)

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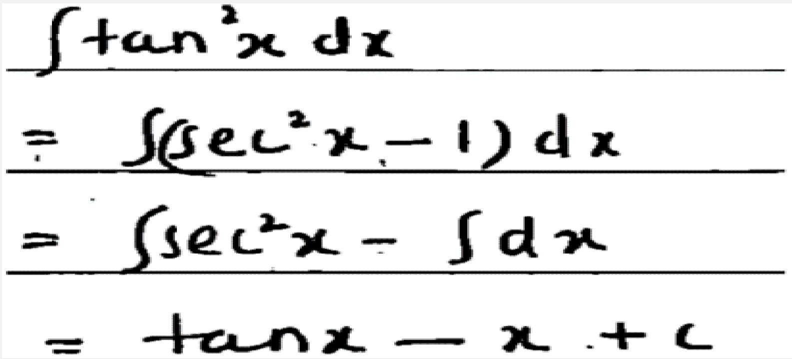
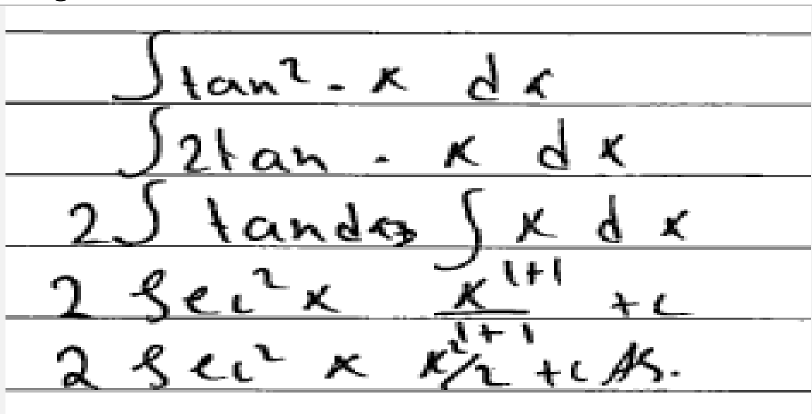
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
While teaching these concepts, teachers are advised to use the following strategies.
Use visual aids like graphs and diagrams to enhance understanding.
Derive equations step by step using a general point on the curve and focus on slope relationships
Explain the condition for the tangent's slope to match a given line's slope.
Highlight the role of derivatives in determining slopes and rates of change.
Foster critical thinking through discussions of real-world applications and limitations

Question No. 3ai Candidates were given the choice to attempt any TWO out of the three questions: 3a, 3b and 3c.	
Question Text	Evaluate the integral $\int \tan^2 x dx$.
SLO No.	16.2.3
SLO Text	Evaluate indefinite integrals.
Max Marks	2
Cognitive Level	A
Checking Hints	1 mark for correct trigonometric substitutions. 1 mark for evaluating integral.
Overall Performance	This part of 3a, i.e. 3ai tested candidates integrated trigonometric function of higher power by reducing to single power using related identity. Overall, a lesser number of the candidates attempting this question scored full marks. Most of the candidates solved this question partially correct.
Description of Better Responses	Better responses showed correct substitution for the given function using identity. Furthermore, such responses demonstrated correct integration of the function obtained after using the identity.
Image of Better Response	
Description of Weaker Responses	Weaker responses could not distinguish between the trigonometric function and its angle. Most of them either integrated the trigonometric function and its angle separately one by one or applied trigonometric identity that was not appropriate or helpful for integrating the given function.
Image of weaker Response	
Question No. 3aii	
Question Text	Integrate the function $y'(t) = \frac{\pi}{t^2}\sin\left(\frac{\pi}{t}\right)$ by substitution.
SLO No.	16.3.2
SLO Text	Evaluate indefinite integrals using appropriate substitutions
Max Marks	3
Cognitive Level	A

Checking Hints	1 mark for writing the function in integral form. 1 mark for making the substitution. 1 mark for evaluating integral.
Overall Performance	This part of 3a, i.e., 3aii tested candidates' ability to integrate a given function by substitution. Moreover, the function was given in derivative form, so it aimed at assessing the candidates' concept of integration as antiderivative. Overall, a lesser number of the candidates attempting this question scored full marks. Most of the candidates solved this question partially correct.
Description of Better Responses	Better responses made the correct substitution for both the angle function and differential (*dx*). Moreover, the candidates integrated by the mentioned method correctly to obtain the correct integral.
Image of Better Response	let $x = \frac{\pi}{t}$ $y'(t) = \frac{\pi}{t} \sin\left(\frac{\pi}{t}\right)$ taking diff on b·s $= -dx \sin x$ $d(x) = d\left(\frac{\pi}{t}\right)$ $= -\sin x\, dx$ $dx = \pi\, d(t)^{-1}$ taking integral $dx = \pi(-t^{-2})$ $-\int \sin x\, dx$ $dx = \frac{-\pi}{t^2}$ $-(-\cos x) + c$ $-dx = \frac{\pi}{t^2}$ $\cos x + c$ $\cos\frac{\pi}{t} + c$
Description of Weaker Responses	Weaker response could not apply substitution method of integration. Most of them either used the formula for integration by parts or integrated both the functions separately.
Images of weaker Responses	**Image (i)** $U = \frac{\pi}{t^2}$ $V = \sin\frac{\pi}{t}$ $\int U.V\, dw = U\int V\, dw - \int\left\{\frac{dU}{dw}\int V\, dw\right\} dw$ $= \frac{\pi}{t^2}\int \sin\frac{\pi}{t}\, dt - \int\left\{\frac{d}{dt}\frac{\pi}{t^2}\int \sin\frac{\pi}{t}\, dt\right\} dt$ $= -\frac{\pi}{t^2}\, \frac{\cos\frac{\pi}{t}}{t} - \int -\frac{\pi}{t^3}\, \frac{\cos\frac{\pi}{t}}{t}\, dt$

	Image (ii) $\int m \cdot \sin m$ By Parts $= U\int v\,dx \cdot \int\left(U' \int v\,dx\right) dx \quad U = m = U' = 1$ $= 1(-\cos m) \cdot \int (1)(-\cos m)\,dx \quad V = \sin m = \int v\,dx = -\cos m$ $= -\cos m \int -\cos m\,dx$ $= -\cos m - \sin m + C$ - eqi Put the value of m in eqi $\boxed{= -\cos\frac{\pi}{t^{+}} - \sin\frac{\pi}{t^{-}} \quad \text{Ans}}$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
While teaching these concepts, teachers are advised to use the following strategies.
Step-by-Step Identity Application: Offer a detailed breakdown of how to apply these identities to simplify higher power trigonometric expressions. Guide candidates through the process with clear, sequential steps.
Algebraic and Graphical Representations: Illustrate concepts with both algebraic and graphical representations. Show how identities are applied algebraically and how they impact the graph.
Revisiting the Concept of Antiderivatives: Reinforce the concept of integration as finding antiderivatives. Highlight the connection between differentiation and integration.
Diverse Integration Scenarios: Encourage candidates to work through various integration scenarios using substitution. Present problems with different levels of complexity and function types.

Question No. 3b Candidates were given the choice to attempt any TWO out of the three questions: 3a, 3b and 3c.	
Question Text	Evaluate the integral $\int (1+x)^2 \ln(1+x)dx$.
SLO No.	16.4.3
SLO Text	Evaluate integrals using integration by parts.
Max Marks	5
Cognitive Level	A
Checking Hints	1 mark for correct selection of $f(x)$ and $g'(x)$. 1 mark for finding $f'(x)$ and $g(x)$. 1 mark for correct substitution. 1 mark for performing the integration. 1 mark for simplification.
Overall Performance	In this question candidates demonstrated their ability to integrate a given function using integration by parts. Only few of the candidates attempting this question scored full marks by showing each and every essential step. However, many of the candidates solved this question partially correct.
Description of Better Responses	Better responses showed good grasp on the concept of integration by parts and applied this method correctly to integrate the function. Such responses did not make any mistake in correct integration of parts.

<table>
<tr><td>Image of Better Response</td><td>Let take: $\ln(1+x) = u$
$(1+x)^2 = v$
By-parts: $u\int v\,dx - \int\{u'\int v\,dx\}\,dx$
For $u' = \frac{1}{1+x}$
$\therefore \ln(1+x)\int(1+x)^2\,dx - \int\{\frac{1}{1+x}\int(1+x)^2\,dx\}$
$\Rightarrow \ln(1+x)\cdot\frac{(1+x)^3}{3} - \int\frac{1}{1+x} \times \frac{(1+x)^3}{3}$
$\Rightarrow \ln(1+x)\cdot\frac{(1+x)^3}{3} - \int\frac{(1+x)^2}{3}$
$\Rightarrow \ln(1+x)\cdot\frac{(1+x)^3}{3} - \frac{1}{3}\int(1+x)^2$
$\Rightarrow \ln(1+x)\cdot\frac{(1+x)^3}{3} - \frac{1}{3}\cdot\frac{(1+x)^3}{3}$
$\Rightarrow \ln(1+x)\cdot\frac{(1+x)^3}{3} - \frac{(1+x)^3}{9}$</td></tr>
<tr><td>Description of Weaker Responses</td><td>Weaker responses could not apply the method of integration by parts correctly. Most of them took the 1st function as differentiable, which is incorrect. Others integrated separately, which was also incorrect.</td></tr>
<tr><td>Image of weaker Response</td><td>$y = \int(1+x)^2\int\ln(1+x)\,dx.$
$f(x) = (1+x)^2\frac{d}{dx}\ln(1+x) - \int\frac{d}{dx}(1+x^2)\ln(1+x)$
$= (1+x^2)\cdot\frac{1}{\cancel{(1+x)}} - \int(2x)\cdot\ln(1+x)$
$= (1+x) - 2\left(\frac{x^2}{2}\cdot\frac{\ln(1+x)^2}{2}\right)$
$1+x - \cancel{2}\left(\frac{x^2}{\cancel{2}}\right)\cdot\frac{\ln(1+x)^2}{2}.$
$1+x - x^2 \times \frac{1}{2}\ln(1+x)^2$
$2(1+x-x^2)\cdot\ln(1+x)^2$
$2+2x-2x^2 \quad \ln(1+x)^2$
$2(1+x \div x^2)\cdot\ln(1+x)^2$</td></tr>
</table>

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
While teaching the concept, teachers are advised to use the following strategies.
Common Trigonometric Integrals: Address common integrals involving trigonometric functions using integration by parts. Offer strategies for simplifying trigonometric integrals.
Integration by Parts vs. Other Techniques: Discuss scenarios where integration by parts is particularly useful compared to other integration techniques. Contrast it with substitution and other methods to guide when to choose integration by parts.
Choice of Functions (u and dv): Emphasise the importance of strategically selecting the "u" and "dv" components to simplify the integration. Offer guidelines such as the "LIATE" rule to aid in making effective choices.
Practice with Different Function Types: Offer a diverse set of practice problems that encompass a variety of function types. This ensures that candidates are comfortable applying integration by parts to different scenarios.

Question No. 3ci

Candidates were given the choice to attempt any TWO out of the three questions: 3a, 3b and 3c.

Question Text	Calculate the area bounded by the function $y(t)=\cos\left(\frac{t}{2}\right)$ between $t=0$ to $t=\frac{\pi}{2}$.
SLO No.	16.6.5
SLO Text	Calculate the area under the curve using definite integrals.
Max Marks	2

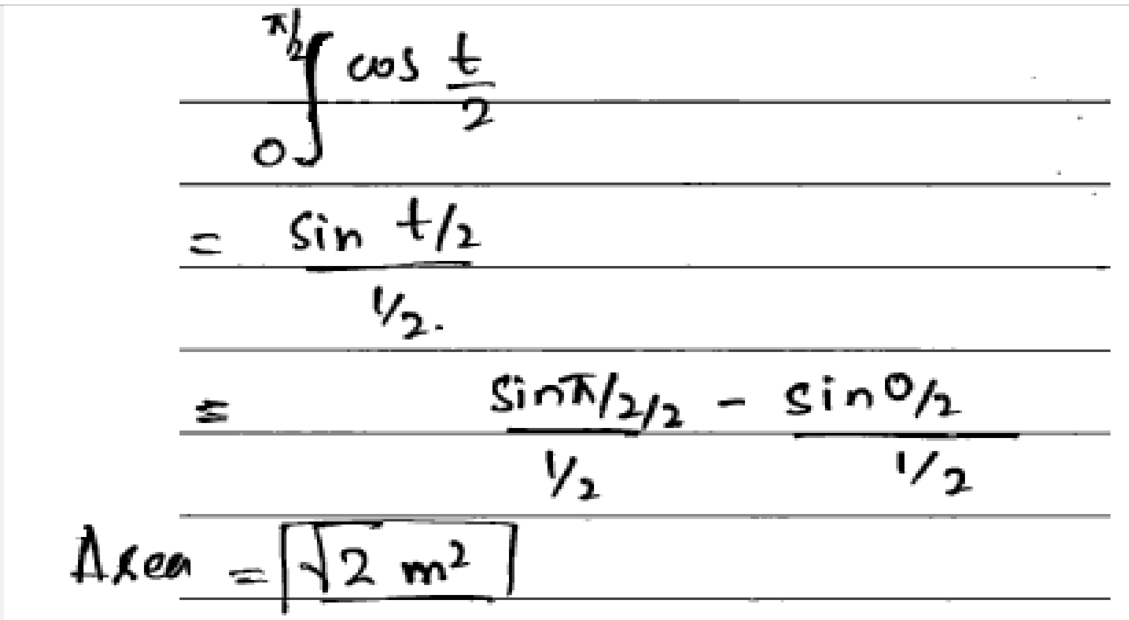
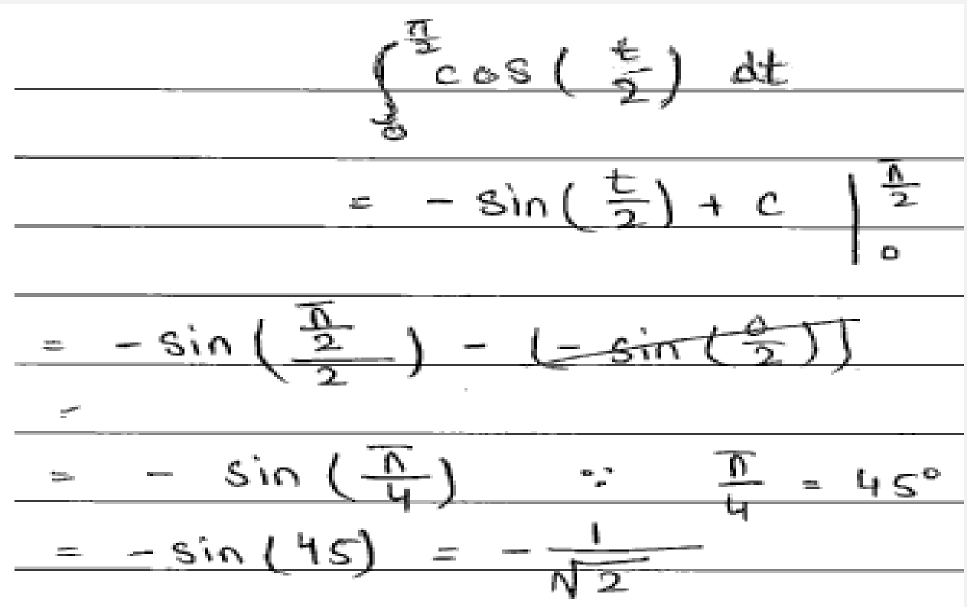
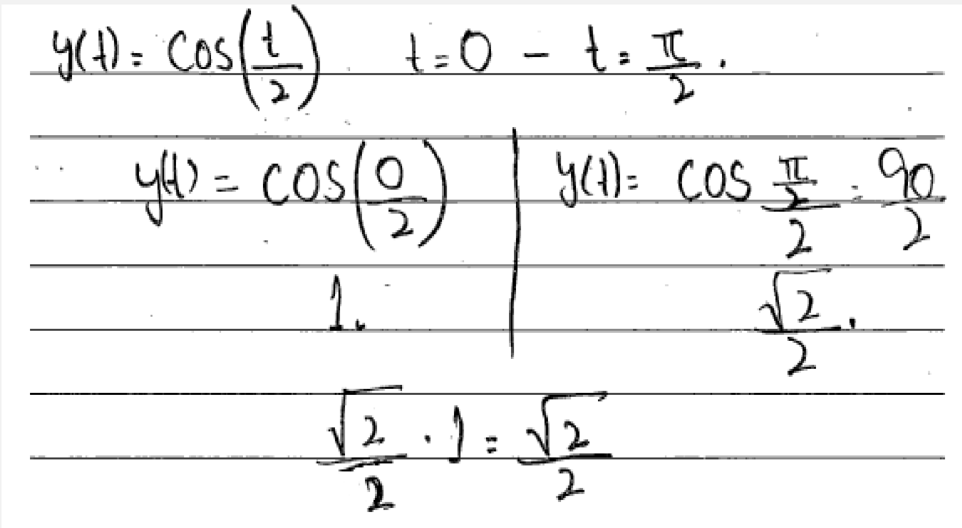
Cognitive Level	A
Checking Hints	1 mark for writing the integral. 1 mark for evaluating area by substituting the limits of integration.
Overall Performance	A considerable number of candidates attempted 3c. This question tested candidates' ability to apply the definite integrals for finding the area under the curve. Overall, good number of the candidates attempting this question scored full marks. However, some of the candidates solved this question partially correct.
Description of Better Responses	Better responses demonstrated good understanding of the area under the curve as definite integral. Therefore, they applied the definite integral and evaluated the value correctly.
Image of Better Response	
Description of Weaker Responses	Most of the weaker responses demonstrated some knowledge of area under the curve as definite integral, however, such responses could not evaluate the integral correctly. In addition, some of the candidates either applied limits without integration of the given function or made mistakes in applying limits of integration.
Images of weaker Responses	**Image(i)** 

	Image (ii)  $y(t) = \cos\left(\frac{t}{2}\right)$ $t = 0 - t = \frac{\pi}{2}$. $y(t) = \cos\left(\frac{0}{2}\right)$ | $y(t) = \cos \frac{\frac{\pi}{2}}{2} = \frac{90}{2}$ $1.$ | $\frac{\sqrt{2}}{2}$. $\frac{\sqrt{2}}{2} \cdot 1 = \frac{\sqrt{2}}{2}$

Question No. 3cii	
Question Text	Show that the general solution of the differential equation $\left(x^2-1\right)\frac{dy}{dx}+x\left(y+1\right)=0$ is $\ln\left(y+1\right)+\frac{1}{2}\ln\left(x^2-1\right)=C$.
SLO No.	16.7.2
SLO Text	Solve differential equations of first order and first degree by separating the variables.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for separating variables 2 marks for applying integral (1 for each on LHS and RHS)
Overall Performance	A considerable number of candidates attempted 3c. This required to solve a given differential equation. Overall, a good number of the candidates attempting this question scored full marks. However, some of the candidates solved this question partially correct.
Description of Better Responses	Better responses demonstrated the complete solution that included separation of the expressions in terms of *x* and *y* followed by integration. Moreover, some candidates differentiated the given solution and substituted in the given differential equation to prove that left hand side of the equation is equal to right hand side. Since both methods are correct, both the solutions that were correctly executed, are considered as better responses.

Image of Better Response	$(x^2-1)\left(\frac{dy}{dx}\right) + x(y+1) = 0$ $(x^2-1)\frac{dy}{dx} = -x(y+1)$ $\frac{1}{y+1}\frac{dy}{dx} = \frac{-x}{x^2-1}$ $\frac{1}{y+1}dy = \frac{-x}{x^2-1}dx$ apply integral on b.s $\int \frac{1}{(y+1)}dy = \int \frac{-x}{x^2-1}dx$ according to $\int \frac{f'(x)}{f(x)}dx = \ln(f(x))$ $\ln(y+1)+c_1 = -\frac{1}{2}\int \frac{2x}{x^2-1}dx$ $\ln(y+1)+c_1 = -\frac{1}{2}\ln(x^2-1)+c_2$ $\ln(y+1)+\frac{1}{2}\ln(x^2-1) = c_2-c_1$ $\ln(y+1)+\frac{1}{2}\ln(x^2-1) = C$ $\boxed{C = c_2 - c_1}$ proved
Description of Weaker Responses	Weaker responses demonstrated insufficient understating of the differential equations and their solution. Most of the candidates with weaker responses either could not separate the variables properly before integrating or they were not familiar with integration of the function.
Images of weaker Responses	**Image (i)** $\frac{dy}{dx} = \frac{-x(y+1)}{(x^2-1)}$ $\frac{(x^2-1)(-1)\frac{dy}{dx} - (-x(y+1))(2x)}{(x^2-1)^2}$ $\frac{\cancel{(x^2-1)}(-1)\frac{dy}{dx} - (-x(y+1))(2x)}{(x^2-1)^{\cancel{2}}}$ $\frac{-\frac{dy}{dx} + x - (y-1) - 2x}{(x^2-1)}$

	Image (ii)
	$(x^2-1)\dfrac{dy}{dx} + x(y+1) = 0$ $(x^2-1)dy = -x(y+1)dx$ Integrating both sides. $\int (x^2-1)dy = -\int (y+1)\,\cancel{dy}\,dx$ $x^2-1 = -(y+1)$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Question Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login
Any Additional Suggestion: Teacher may use these teaching strategies, support students in mastering both differential equations and the evaluation of definite integrals for area calculations, enabling them to excel in these fundamental calculus concepts. Visual Aids: Use visual aids, diagrams, and graphs to visually represent concepts and solutions, aiding in comprehension. Error Analysis: Address common mistakes and misconceptions that students might encounter in both topics and provide guidance on how to avoid them. Challenge: Assign progressively more challenging problems that require critical thinking and creative problem-solving skills, reinforcing the application of concepts.		

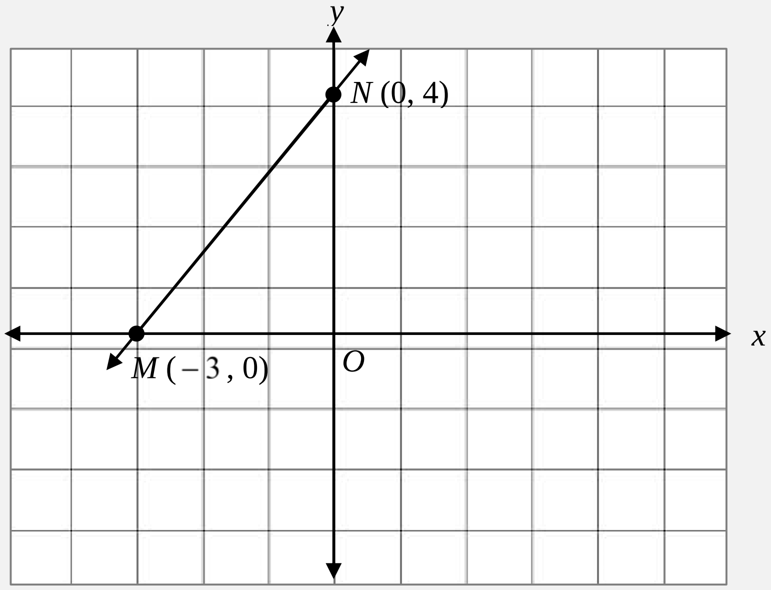
Question No. 4	
Question Text	In the given figure, the line passes through two given points $M(-3, 0)$ and $N(0, 4)$.  Using the given figure, i. find the slope of the given line. ii. show that equation of the line is $4x-3y+12=0$.
SLO No.	17.2.3, 17.4.3
SLO Text	Find the slope of a line passing through two points. Convert the general form of the equation of a straight line into the forms mentioned in SLO 17.4.2.
Max Marks	4
Cognitive Level	A
Checking Hints	i. 1 mark for writing the correct values in the slope formula. 1 mark for evaluating slope correctly. ii. 1 mark for substituting the given values in the two-point form 1 mark for simplification.
Overall Performance	This question had two parts. In the first part, candidates calculated the slope of the line with the help of two given points on the line. In the second part, candidates found the equation of the line as stated in the question. Overall, majority of the candidates attempted this question correctly and completely scoring full marks. However, some of the candidates solved this question partially correct.
Description of Better Responses	Better responses used the formula for slope of a line correctly and evaluated slope without any substitution or calculation error. In the second part, candidates giving better responses found the equation of the line through multiple methods, i.e., point slope form, two-point form and determinant form. Some better responses are shown as under. Most of the candidates applied the method used in the example.

Image of Better Response	$M(-3,0)$; $N(0,4)$ $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{4-0}{0-(-3)} = \dfrac{4}{3}$ Slope $= m = \dfrac{4}{3}$ Ans ii. show that equation of the line is $4x-3y+12=0$. (2 $y-y_1 = \dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$ $\overset{x_1\ y_1}{M(-3,0)}$; $\overset{x_2\ y_2}{N(0,4)}$ $(y-0) = \dfrac{4-0}{0+3}(x+3)$ $3(y) = 4(x+3)$ $3y = 4x+12 \longrightarrow 4x-3y+12=0$
Description of Weaker Responses	In the first part, candidates applied incorrect formula for finding the slope, as shown in the given example. In addition, there were few responses which reflected that candidates did not have the concept of slope altogether although the formula was given in the formula sheet. It is advisable to incorporate formula sheets in the internal examinations in school, so that candidates would practice the effective use during examinations. In the second part, candidates were unable to understand the requirement of the question, hence calculated some irrelevant value.
Images of weaker Responses	**Image (i)** i. $\dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$ | $\dfrac{4}{3}\times 3 = 4$ $\dfrac{(y-0)}{(0-3)}(0-(-3))$ | Slope of the given lini is 4 $\dfrac{4}{3}(3)$ ii. $d = \dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ | $\dfrac{4x+3y+12}{\sqrt{16+9}}$ $4(-3)+3(0)+$ | $\dfrac{4x+3y+12}{5}$ | $4x+3y+12=5$

Image (ii)

i.

slope of line $\overline{MN}$ is $45°$.

as point N is on axis $x = 0,\ y = 4$.

and point M is on axis $x = -3,\ y = 0$

ii.

$4x - 3y + 12 = 0$

$(4y)(0x) - (3x)(0y) + 12 = 0$.

$(4y)(0y) - (3x)(0x) + 12 = 0$

$y - x + 12 = 0$.

Suggestions for Improvement (Highlighted part)

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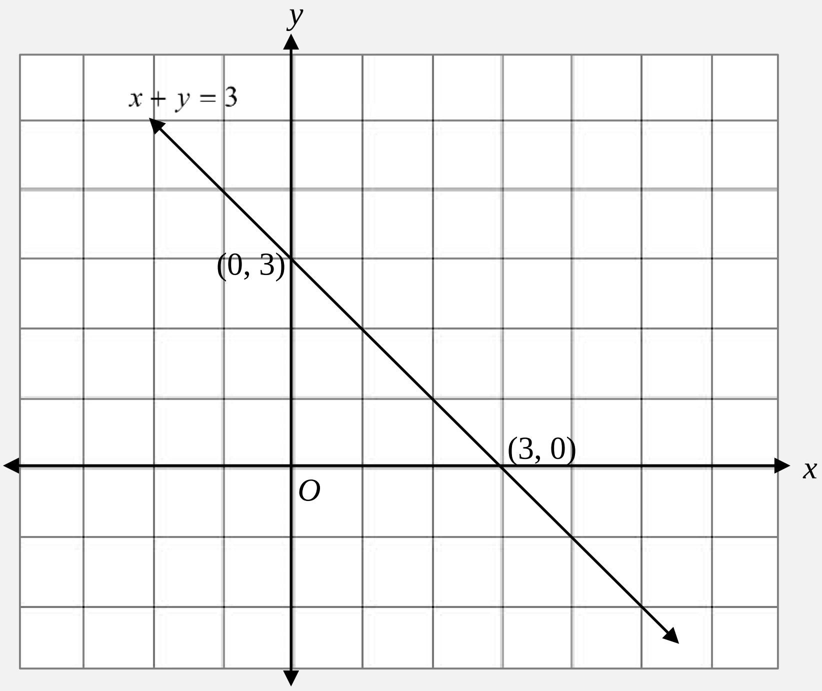
Any Additional Suggestion:
Following are some interactive teaching strategies to help students understand how to find the slope of a line through two points:
Real-world Examples: Use real-world scenarios where students have to find the slope. For instance, discuss scenarios like calculating the slope of a hill or a ski slope, emphasizing that slope measures steepness.
Physical Manipulatives: Provide students with physical objects like wooden blocks or toy cars and ask them to create inclined planes. They can then measure the rise and run to calculate the slope.
Interactive Whiteboard: Use an interactive whiteboard to draw lines through points, and allow students to manipulate the points and observe how the slope changes in real-time
Visual Representations: Use visual aids like diagrams and graphs to illustrate how the different forms of the equation relate to the geometry of the line.
Step-by-Step Examples: Provide step-by-step examples and ask students to follow along. Encourage them to work through practice problems to reinforce the concepts.
Discussion and Peer Teaching: Foster class discussions where students explain their thought processes when converting equations. Encourage peer teaching, where students explain the conversions to their classmates.

Question No. 5	
Question Text	Using the given graph, • draw the line $2x - y = 0$. • find the maximum value of the function $f(x, y) = 2x + y$ subject to the following constraints. $2x - y \leq 0$ $x + y \leq 3$ $x \geq 0,\ y \geq 0$ 
SLO No.	18.4.3
SLO Text	Solve simple linear programing problems.
Max Marks	4
Cognitive Level	A

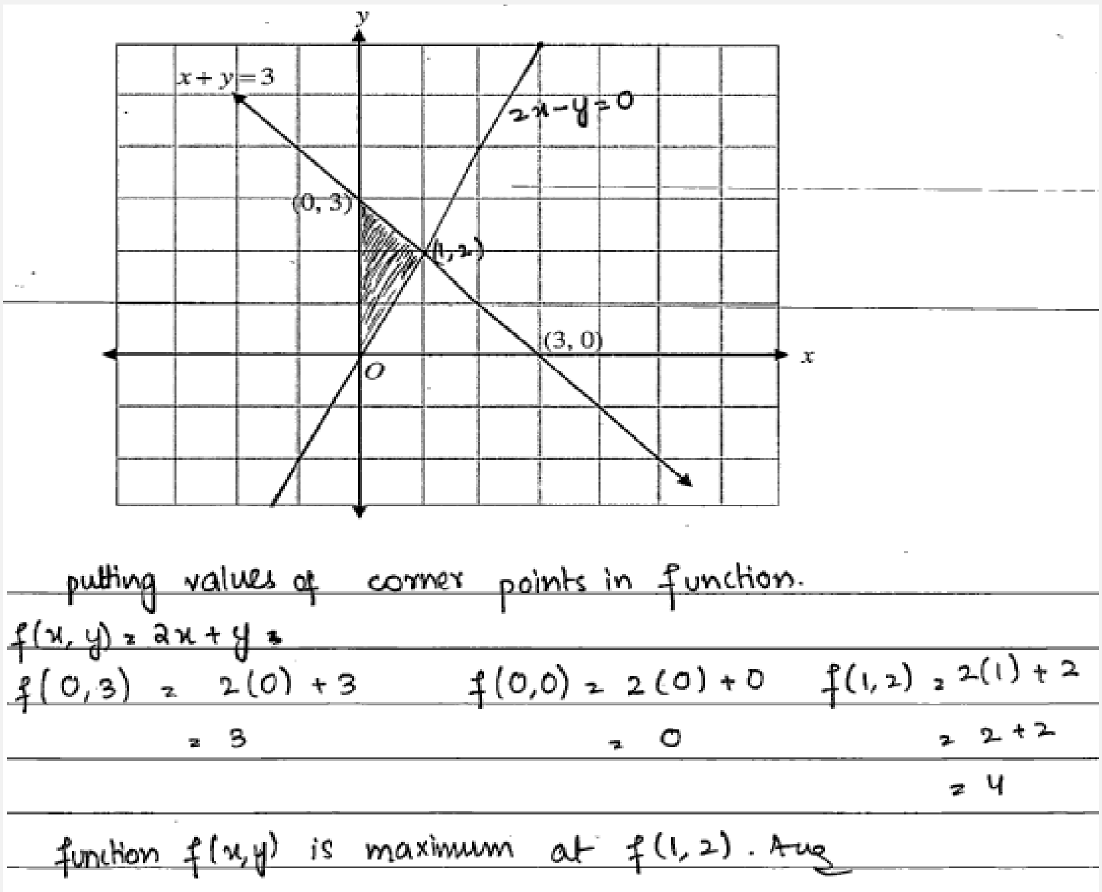
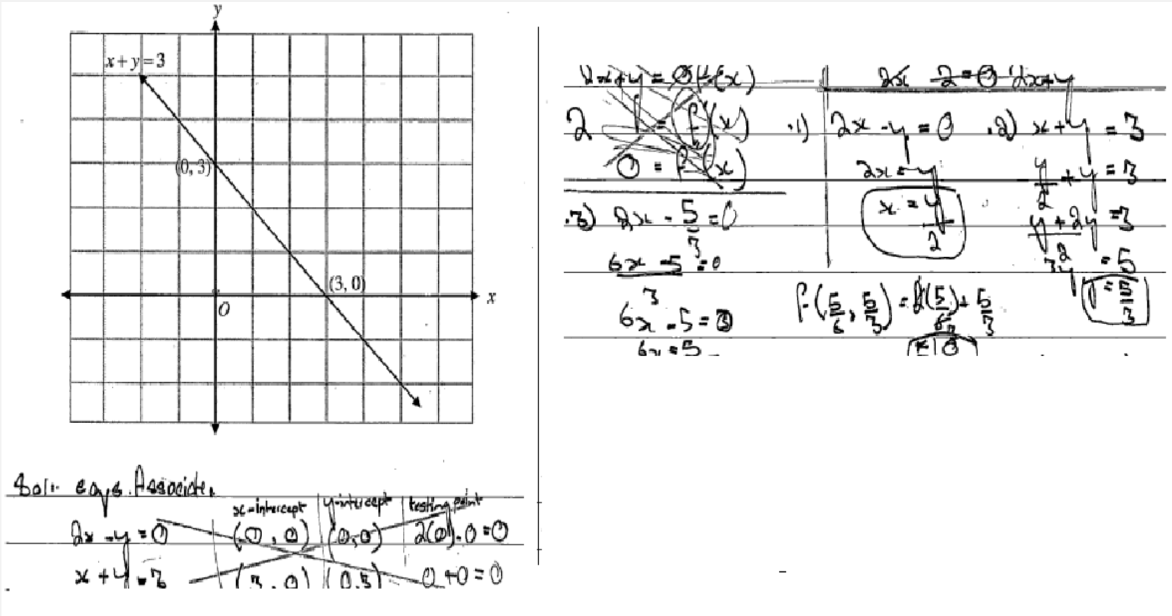
Checking Hints	1 mark for drawing the line representing the inequality. 1 mark for identifying the solution region. 1 mark for identifying the corner point (1, 2). 1 mark for evaluating the maximum value of the objective function.
Overall Performance	This question was related to solving linear programming problem. One of the lines was already drawn on the grid paper. Candidates were required to draw the second line using the constraints, identify and shade the feasible region, identify the corner points then, find the maximum value of the function. Overall, candidates struggled in solving this question. Most of the candidates could either solve this question partially or could not solve at all.
Description of Better Responses	Better responses showed the line accurately drawn with the help of the constraint, shaded the feasible region, identified the corner points and found the ordered pair for which function has maximum value. Such responses did not bear any errors in either calculation or graphical presentation.
Image of Better Response	
Description of Weaker Responses	Weaker responses made errors in either in one or in multiple steps described in the better responses. Some candidates got confused with the partial solution provided to reduce candidates' work, as such questions are not present in the textbooks.

Image of weaker Response	

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
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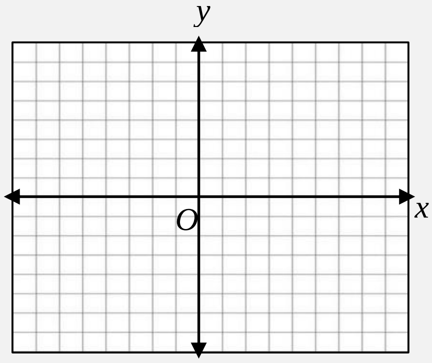
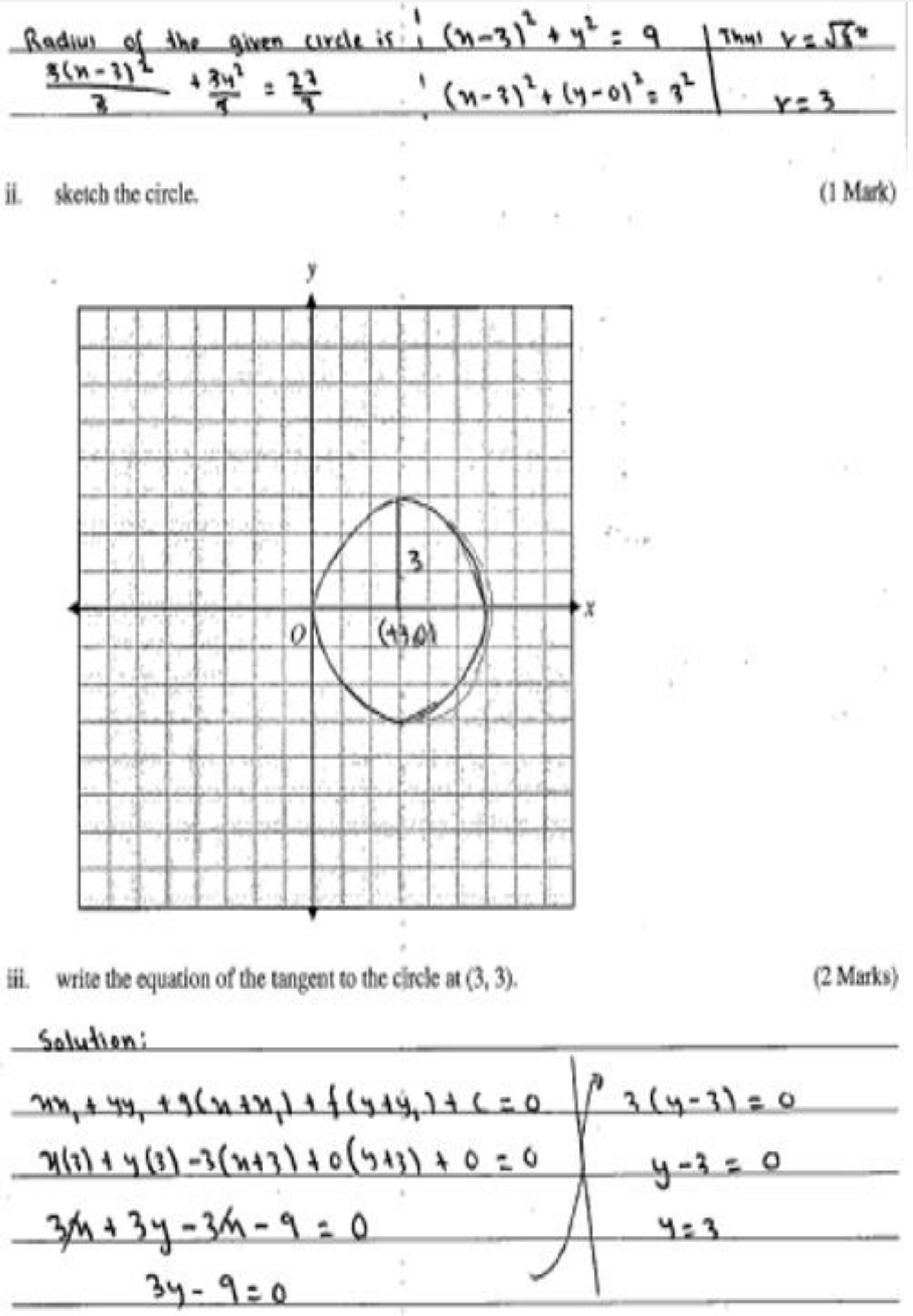
Any Additional Suggestion: While teaching the concept, teachers are advised to use the following strategies. Incorporate real-life examples: Use case studies and real-world scenarios to demonstrate how linear programming is applied in various industries and problem-solving situations. Engage students in problem-solving exercises that involve linear programming. These exercises will help students apply their knowledge and develop critical thinking skills		

Provide detailed explanations: Describe the method in detail and provide step-by-step instructions for solving linear programming problems. This will help learners understand the concepts and techniques involved

Use visual aids: Utilise graphs, charts, and diagrams to visually represent linear programming problems. Visual aids can enhance understanding and make complex concepts more accessible to learners.

Practice projects: Assign practical projects that require students to apply linear programming techniques to solve real-world problems. This will help learners gain hands-on experience and reinforce their understanding

Question No. 6	
Question Text	The equation of a circle is given by $3(x-3)^2+3y^2=27$. For the given circle, i. find the radius. ii. sketch the circle. iii. write the equation of the tangent to the circle at (3, 3). 
SLO No.	19.2.4/19.2.5/19.3.3
SLO Text	Find the centre and radius by using the general and standard form of equation of a circle. Sketch a circle when its elements are given. Find the equation of a tangent to a circle in the slope-intercept form.
Max Marks	4
Cognitive Level	A
Checking Hints	i. 1 mark for calculating the radius ii. 1 mark for drawing the graph iii. 1 mark for finding the slope of the tangent line at (3, 3) 1 mark for the equation of the tangent
Overall Performance	This question was related to finding the centre and radius by using the general and standard form of equation of a circle and sketching it when elements are given. Moreover, it involves finding the tangent of the circle. Overall, candidates struggled in solving this question. Most of the candidates could either solve this question partially or could not solve at all.
Description of Better Responses	Better responses reduced the equation by dividing by 3 on both sides and converting to standard form. Hence, such responses compared the given equation with standard equation of a circle and found radius. With the help of radius and centre depicted from the standard equation of the circle, candidates sketched the circle. Moreover, they either used the direct formula or used differentiation approach to find the equation of the tangent to the circle. Refer to the given example for one of the better response.

<table>
<tr>
<td>Image of Better Response</td>
<td>


Radius of the given circle is : $(x-3)^2 + y^2 = 9$ | Thus $r = \sqrt{9}$

$\frac{3(x-3)^2}{3} + \frac{3y^2}{3} = \frac{27}{3}$; $(x-3)^2 + (y-0)^2 = 3^2$ | $r = 3$

ii. sketch the circle. (1 Mark)

iii. write the equation of the tangent to the circle at (3, 3). (2 Marks)

Solution:

$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

$x(3) + y(3) - 3(x+3) + 0(y+3) + 0 = 0$

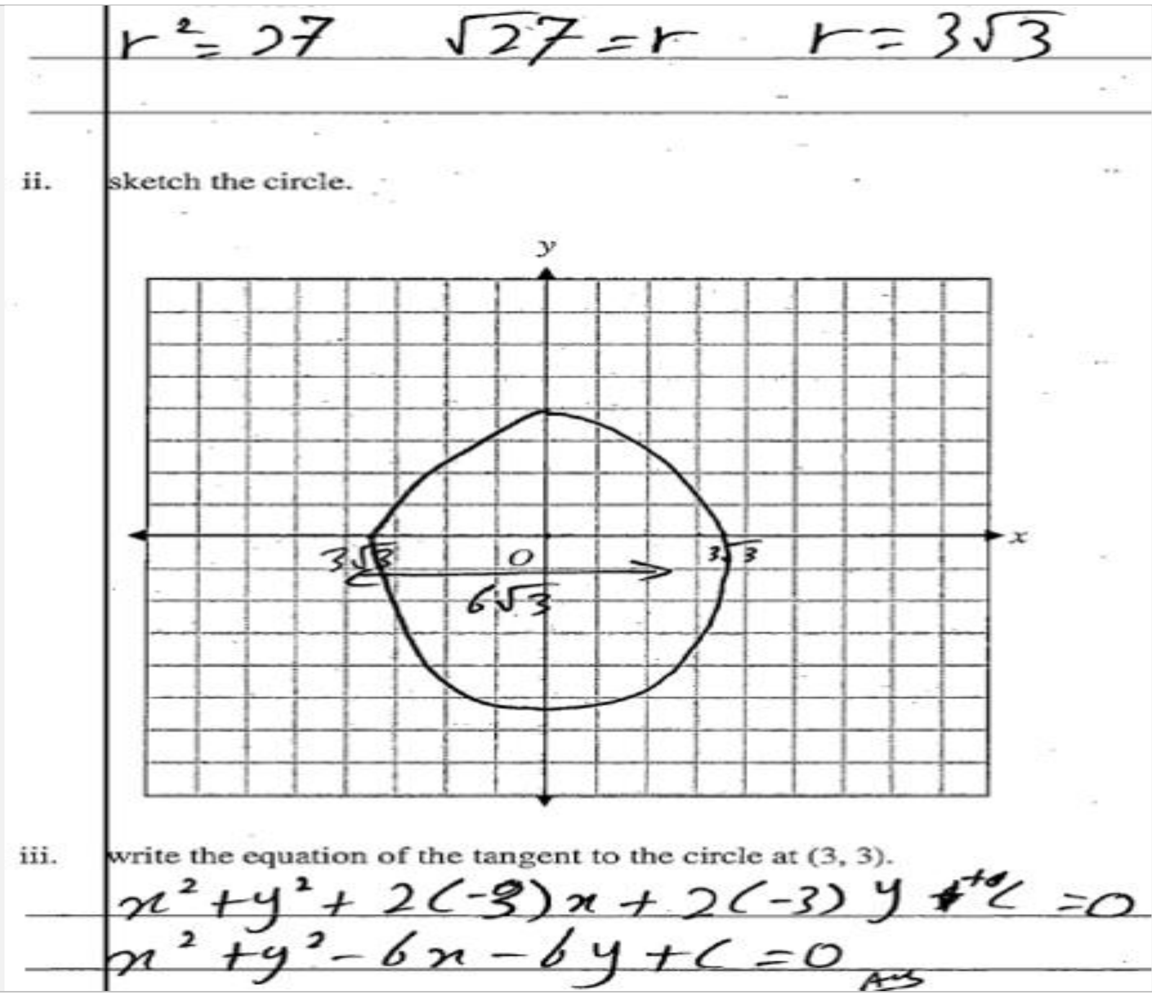
$3x + 3y - 3x - 9 = 0$

$3y - 9 = 0$

$3(y-3) = 0$

$y - 3 = 0$

$y = 3$
</td>
</tr>
<tr>
<td>Description of Weaker Responses</td>
<td>Weaker responses demonstrated the inability to convert the given equation into standard form and extract the necessary elements like the centre of the circle and radius. Hence, such responses could not use the essential elements to sketch the circle and find the equation to</td>
</tr>
</table>

	the tangent at the given point. To solve such questions, candidates must be able to extract of the essential elements of a circle.
Image of weaker Response	

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:
While teaching the concept, teachers are advised to use the following strategies.
Real-Life Examples: Provide real-life scenarios where the conversion of equations into standard form and extraction of essential elements are applicable. For example, demonstrate how architects use these concepts to design circular structures.
Activity-Based Learning: Engage students in hands-on activities that involve converting equations into standard form and identifying essential elements. For instance, provide worksheets or interactive exercises where students can practice identifying the centre and radius of circles.
Visual Aids: Utilise visual aids such as diagrams, graphs, and illustrations to illustrate the conversion process and identify essential elements. Visual representations can help students visualise the concepts and make connections between different elements.
With regular opportunities to practice and reinforce their understanding, learners may develop the necessary skills and confidence. Assigning practice projects that simulate real-world scenarios can greatly enhance their learning experience.

Question No. 7a Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.	
Question Text	Find the equations of tangents to the parabola $y^2 = 8x$, at the points whose *x*–coordinate is 2.
SLO No.	20.3.2
SLO Text	Find the equation of a tangent and a normal to a parabola. i. at a point, ii. which is parallel to a line, iii. which is perpendicular to a line.
Max Marks	4
Cognitive Level	A
Checking Hints	1 mark for finding points (2, 4) and (2, – 4) 1 mark for finding the slopes of tangents 1 mark for writing the point slope form of the equation 1 mark for finding the equations of the tangents
Overall Performance	This question tested the essential learning outcomes related to parabola. Candidates were required to find the points on the parabola using the given values. Moreover, the tangent at those points were also to be found. Overall, though lesser number of candidates attempted this question, majority of the candidates attempting this question were not able to score full marks. Most of the candidates solved this question partially correct.
Description of Better Responses	Better responses showed correct calculation of the required points; followed by calculation of slopes of the tangents at the given points and found the equation of the tangents at those points.

Image of Better Response	We first have to find y coordinates of tangents. $y^2 = 8(2)$ $y^2 = 16$ $y = \pm 4$ We have tangents at point $(2,4)$ and $(2,-4)$ Now we find slope :- $\frac{d}{dx} y^2 = \frac{d}{dx} 8x$ $= 2y \cdot \frac{dy}{dx} = 8$ $\boxed{\frac{dy}{dx} = \frac{4}{y}}$ At point $(2,4)$ $m = \frac{4}{4}$ $m = 1$ $y - y_1 = m(x - x_1)$ $y - 4 = 1(x - 2)$ $y - 4 = x - 2$ $y - x - 2 = 0$ $\boxed{y - x - 2 = 0}$ First tangent At point $(2,-4)$ $m = \frac{4}{-4}$ $m = -1$ $y + 4 = -1(x - 2)$ $y + 4 = -x + 2$ $y + x + 2 = 0$ $\boxed{y + x + 2 = 0}$ second tangent
Description of Weaker Responses	Weaker responses reflected that candidates could not easily determine what was to be evaluated. However, some of the candidates evaluated a single value of the *y* coordinate corresponding to *x*. In addition, most of such responses could not demonstrate how to find the equation of the tangents at that/ those points.
Image of weaker Response	Sol: $y^2 = 4ax \Rightarrow y^2 = 4(2)x$ Put $x = 2$ $y^2 = 4(2)(2)$ $y^2 = 16$ $\sqrt{y^2} = \sqrt{16}$ $y = 4$ Now, Put $x = 2$ & $y = 4$ $\Rightarrow (y-k)^2 = 4a(x-h)$ $\Rightarrow (4-k)^2 = 4(2)(2-h)$ $\Rightarrow 16 - k = 8(2-h)$ $\Rightarrow 16 - k = 16 - h$ $\Rightarrow -k = -h$ $\Rightarrow k = h$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

Teachers are suggested to use a combination of visual aids, hands-on activities, and technology to make the learning of conic sections engaging and interactive:

Interactive Graphing Software: Utilise graphing software like GeoGebra or Desmos to plot parabolas and their tangent lines. These tools allow students to manipulate the parameters and observe how the tangent changes as they move along the curve.

Real-world Examples: Show real-world examples of parabolic shapes, such as the path of a thrown object or the reflector in a flashlight. Discuss how tangents are relevant in these contexts.

Tangent Line Demonstrations: Physically demonstrate the concept of a tangent line by drawing a curved path on the ground and showing how a straight object (like a stick) touches the curve at one point without crossing it.

Exploration of Slope: Help students understand that the slope of the tangent line at any point on the parabola is equal to the derivative of the parabola's equation at that point. Walk them through the process of finding the slope of the tangent at various points.

Interactive Worksheets: Create worksheets or online activities where students can practice finding tangent lines to parabolas at specific points. Provide instant feedback to reinforce their understanding.

Visual Demonstrations: Use physical props like a flashlight and a concave mirror to show how light rays reflect and create a parabolic shape. Discuss how tangent lines are related to the angle of reflection.

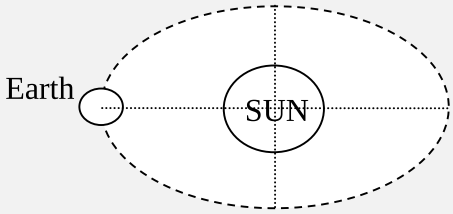
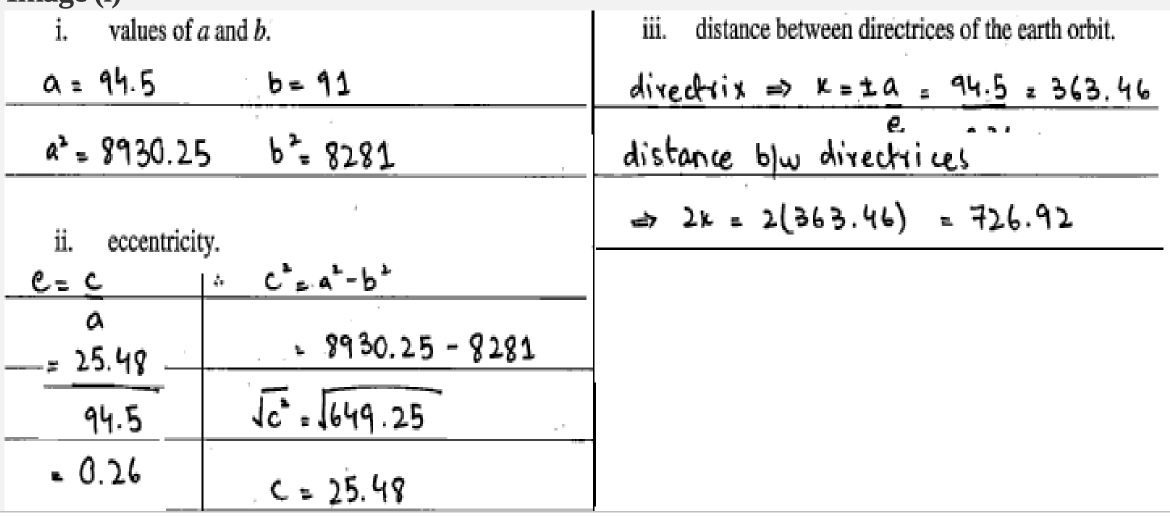
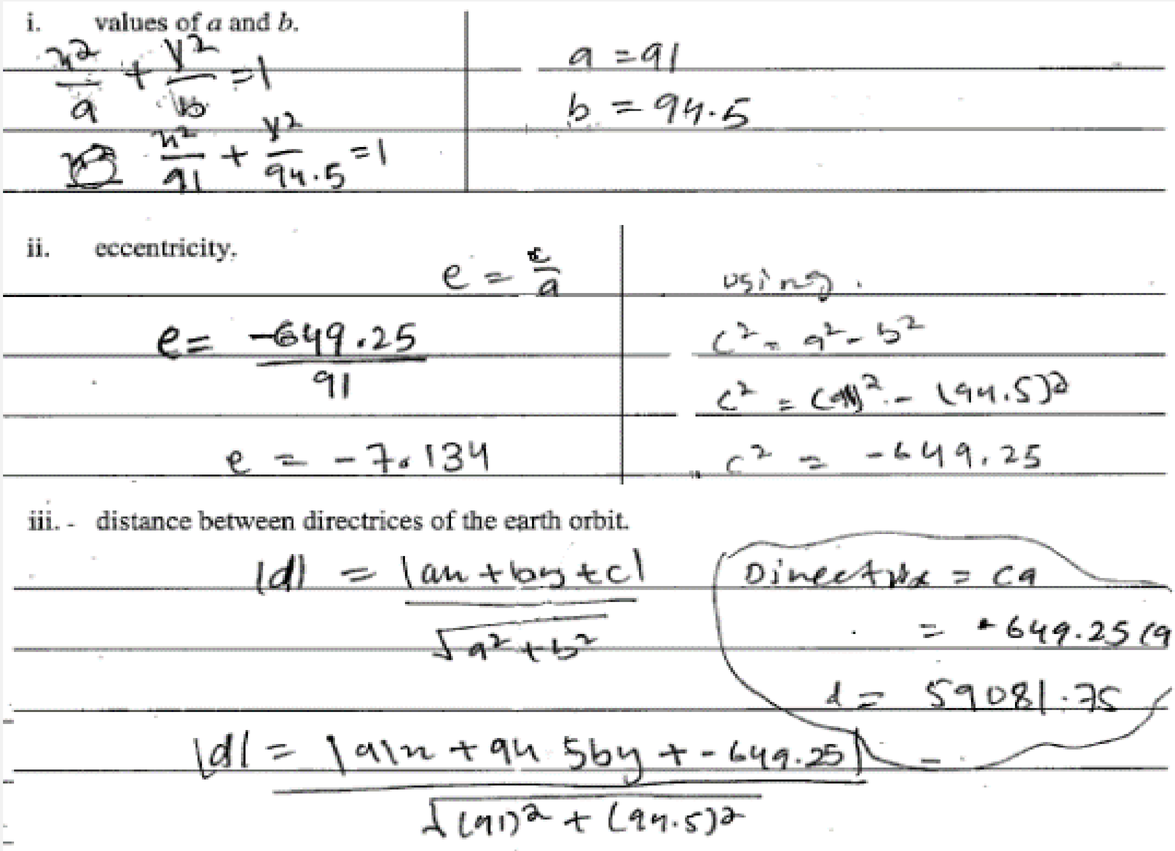
Question No. 7b Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.	
Question Text	The earth is moving around the sun in an ELLIPTICAL path. During this motion, the shortest distance between their centers is 91 million miles while the farthest is 94.5 million miles. If origin is considered at the centre of the sun, then equation of this ellipse will have the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the i. values of *a* and *b*. ii. eccentricity. iv. distance between directrices of the earth orbit. 
SLO No.	21.2.5
SLO Text	Solve word problems related to ellipse.
Max Marks	4
Cognitive Level	A
Checking Hints	i. 1 Mark for correct identification of *a* and *b* ii. 1 Mark for value of *c* 1 Mark for eccentricity 1 Mark for calculating the distance between directrices
Overall Performance	This question was related to application of ellipse. Candidates were required to find the equation for the ellipse using the given information. Moreover, candidates were expected to calculate the listed elements of the ellipse. Overall, a good number of candidates attempted this question, many of the candidates attempting this question scored full marks. The remaining candidates solved this question partially correct.
Description of Better Responses	Better responses used the given information to form the equation of the ellipse in standard form. Moreover, they evaluated the required elements of the ellipse using the equation. Hence, candidates could form the equation from the given situation and they were also able to relate the elements with the context in which this problem was posed.
Images of Better Responses	**Image (i)**  i. values of *a* and *b*. $a = 94.5$ $b = 91$ $a^2 = 8930.25$ $b^2 = 8281$ ii. eccentricity. $e = \frac{c}{a}$ ∴ $c^2 = a^2 - b^2$ $= 8930.25 - 8281$ $= \frac{25.48}{94.5}$ $\sqrt{c^2} = \sqrt{649.25}$ $= 0.26$ $c = 25.48$ iii. distance between directrices of the earth orbit. directrix $\Rightarrow x = \pm\frac{a}{e} = \underline{94.5} = 363.46$ distance b/w directrices $\Rightarrow 2x = 2(363.46) = 726.92$

	Image (ii) $a=94.5$ $b=91$ eccentricity. $\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{94.5^2-91^2}{94.5}}$ ecentricity $= \frac{\sqrt{53}}{27}$ distance between directrices of the earth orbit. $\cancel{\pm a/e}$ $\frac{2a}{e}$ $\frac{2(94.5)}{\sqrt{53}/27} = 700.95$
Description of Weaker Responses	Although candidates with weaker responses were able to recognise that the given situation depicts an ellipse, however, they could not pick the required information to construct the correct equation. Hence, they could not find the required elements. On the contrary, some of the candidates applied incorrect formulae to find the required elements.
Images of weaker Responses	**Image (i)** $x^2/91 - y^2/94.5$ ii. eccentricity. $e = c/a$ $e = \frac{742/2}{91} = c = 53/13$ iii. distance between directrices of the earth orbit. $2a$ $2(91)$ 181

Image (ii)



i. values of a and b.

$\frac{x^2}{a} + \frac{y^2}{b} = 1$

$\frac{x^2}{91} + \frac{y^2}{94.5} = 1$

$a = 91$

$b = 94.5$

ii. eccentricity.

$e = \frac{c}{a}$

$e = \frac{-649.25}{91}$

$e = -7.134$

using:

$c^2 = a^2 - b^2$

$c^2 = (91)^2 - (94.5)^2$

$c^2 = -649.25$

iii. distance between directrices of the earth orbit.

$|d| = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$|d| = \frac{|91x + 94.5y + -649.25|}{\sqrt{(91)^2 + (94.5)^2}}$

Directrix $= ca$

$= -649.25(9$

$d = 59081.75$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion: Teachers are suggested to use a combination of visual aids, hands-on activities, and technology to make the learning of conic sections engaging and interactive: String and Pins: Use string, two pins, and a pencil to demonstrate how an ellipse can be drawn. Students can take turns creating ellipses and discussing their properties. Geometric Art: Have students create geometric art by drawing ellipses of various sizes and shapes. This can help reinforce the idea of the major and minor axes. Astronomical Applications: Discuss how ellipses are used to describe the orbits of planets around the sun. You can use simulations or animations to illustrate these orbits. Digital Tools: Utilise graphing software like GeoGebra or Desmos to show how changing parameters in the equation of an ellipse affects its shape and orientation. Outdoor Activities: Take the class outdoors to a large open area and have students walk along ellipses drawn on the ground. This can provide a physical, hands-on experience

Question No. 7c Candidates were given the choice to attempt any TWO out of the three questions: 7a, 7b, and 7c.	
Question Text	Find the co-ordinates of foci, equations of directrices and lengths of the latera of the hyperbola $x^2 - y^2 = 9$ .of the earth orbit.
SLO No.	22.3.2
SLO Text	Find the equation of a tangent and a normal to a hyperbola.
Max Marks	4
Cognitive Level	A
Checking Hints	1 mark for finding eccentricity e 1 mark for finding coordinates of focus 1 mark for finding equation of directrices 1 mark for finding length of latera recta
Overall Performance	This question was related to hyperbola. Candidates were required to use the given equation to extract the elements. Moreover, they were expected to evaluate other elements using formulae. Overall, a good number of candidates attempted this question and many of the candidates attempting this question scored full marks. The remaining candidates solved this question partially correct.
Description of Better Responses	Candidates with better responses used the given equation correctly to extract the elements. Moreover, they evaluated other elements using formulae. Hence, they correctly evaluated all the listed elements.

Image of Better Response	$\frac{x^2}{9} - \frac{y^2}{9} = 1$. $a = 3$ $b = 3$ $c^2 = a^2 + b^2$. $c^2 = 9 + 9$ $c^2 = \sqrt{}\ 18$ $\boxed{c = \sqrt{18}}$. Foci $(\pm c, 0)$. $(\pm\sqrt{18}, 0)$. equation of directrices $= \pm \frac{c}{e^2}$. $\therefore e = c/a$. $= \pm \frac{\sqrt{18}}{2}$. $e = \sqrt{18}/3$ $\boxed{e = \sqrt{2}}$. $\boxed{= \pm \frac{3\sqrt{2}}{2}}$. 3/ length of latera rectra $= \left\lvert \frac{2b^2}{a} \right\rvert$ $= \left\lvert \frac{2(3)^2}{3} \right\rvert$ $\boxed{\text{length of latera Recta } 6}$
Description of Weaker Responses	Candidates with weaker responses mostly were able to reduce the equation into standard form, however, they could not extract the elements correctly. In addition, they could not apply the relevant formulae to calculate the required elements correctly.
Images of weaker Responses	**Image (i)** $x^2 - y^2 = 9$ $\frac{x^2}{9} - \frac{y^2}{9} = 1$ $a^2 + b^2 = c^2$ $\frac{x^2}{3^2} - \frac{y^2}{3^2}$ $c^2 = 18$ coordinates of foci $= 2c = 2\sqrt{18} = \pm 8.5$ directrices $= 2b^2/a$ $= 2(1)^2/a$ $= 2$ Latus ractum $= c/a$ $= +9/+1$ $= 9$

	Image (ii) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{9}{9}$ $\frac{x^2}{9} - \frac{y^2}{9} = 1$ foci = $(\pm 9, 0)$ Directrices = [illegible] vertices $(\pm 9, 0)$ co-vertices $(0, \pm 9)$

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion: Teachers are suggested to use a combination of visual aids, hands-on activities, and technology to make the learning of conic sections engaging and interactive. Cutting Cones: Demonstrate hyperbolas by cutting a cone at different angles. This visual representation can help students understand the shape of a hyperbola. Geogebra or Desmos: Use dynamic graphing software to allow students to manipulate parameters in the hyperbola equation and observe how it affects the graph. Real-world Examples: Show examples of hyperbolic shapes in real life, such as satellite dish reflectors or certain types of mirrors. Foci and Directrix: Conduct activities where students locate the foci and directrices of hyperbolas. For example, let them find points where light reflects off mirrors to converge at a single point. Hyperbola Construction: Let students construct a hyperbola using a straightedge and compass, similar to the string and pins method for ellipses. Conics in Navigation: Discuss how hyperbolas were historically used for navigation (hyperbolic navigation). Explore the concept of hyperbolic positioning systems.

Question No. 8	
Question Text	By transforming the equation $x^2+5y^2-2x+10y+5=0$ referred to a new origin and axes remaining parallel to the original axes, the first degree terms are removed. Find the coordinates of the new origin and the transformed equation.
SLO No.	23.1.3
SLO Text	Find the transformed equation by using translation or rotation of axes.
Max Marks	4
Cognitive Level	A
Checking Hints	1 mark for substituting correct equations of transformation 1 mark for simplification to get an equation in terms of X and Y 1 mark for finding h and k by applying the given condition 1 mark for final equation
Overall Performance	This question was related to transformation of a given equation by using translation of axes. Candidates were expected to use the given equations and transform the given equation. Overall, candidates struggled in solving this question. Most of the candidates could either solve this question partially or could not solve it at all.
Description of Better Responses	Better responses showed the translation equations and correct substitution. Moreover, such responses showed the simplification and evaluation of correct values of *h* and *k*, hence the correct transformed equation was obtained.

Image of Better Response	$x = X+h$, $y = Y+k$ Substituting in given equation. $(X+h)^2+5(Y+k)^2-2(X+h)+10(Y+k)+5=0$ $X^2+2Xh+h^2+5Y^2+10Yk+5k^2-2X-2h+10Y+10k+5=0$ $X^2+5Y^2+h^2+5k^2+5+10k-2h+X(2h-2)+Y(10k+10)=0$ • $X(2h-2)=0X$ • $Y(10k+10)=0Y$ $2h-2=0$ $10k+10=0$ $h=1$ $k=-1$ $O'(h,k)=(1,-1)$ (new origin) $X^2+5Y^2+(1)^2+5(-1)^2+5+10(-1)-2(1)+X\overbrace{(2(1)-2)}^{0}+Y\overbrace{(10(-1)+10)}^{0}=0$ $X^2+5Y^2-1=0$ (transformed equation)
Description of Weaker Responses	Weaker responses showed that candidates were not clear about the transformation of equations when translation was required. Such responses showed that candidates linked transformation with slope. Moreover, though such candidates used the transformation equation, however, they could not make the substitution and left simplification incomplete.
Images of weaker Response	**Image (i)** $(x+h)^2+5(y+k)^2-2(x+h)+10(y+k)+5=0$ $(x+h)^2+5(y+k)^2-2x-2h+10y+10k+5=0$ $(x+h)^2+5(y+k)^2=0$ $x=h$, $y=-k$ $(-h, -k)$

Image (ii)

$x^2+5y^2-2x+10y+15=0$

origin

if ~~or~~ old $(g,h) = (1,-5)$

if we swith the line but still is parrallel so the origin will change but slope of line will remain same

if we remove 1^{st} degree term

$x^2+5y^2+15=0$ eq of new line without first degree

$X=x-h$ $Y=y-h$

~~$X=x-1$~~ $X+h=x$ $y=Y+5$

$x=X+1$ $y=y-5$

$x^2+5y^2+2(x-1)+(y-5)+5=0$ eq of new line Ans

Suggestions for Improvement (Highlighted part)

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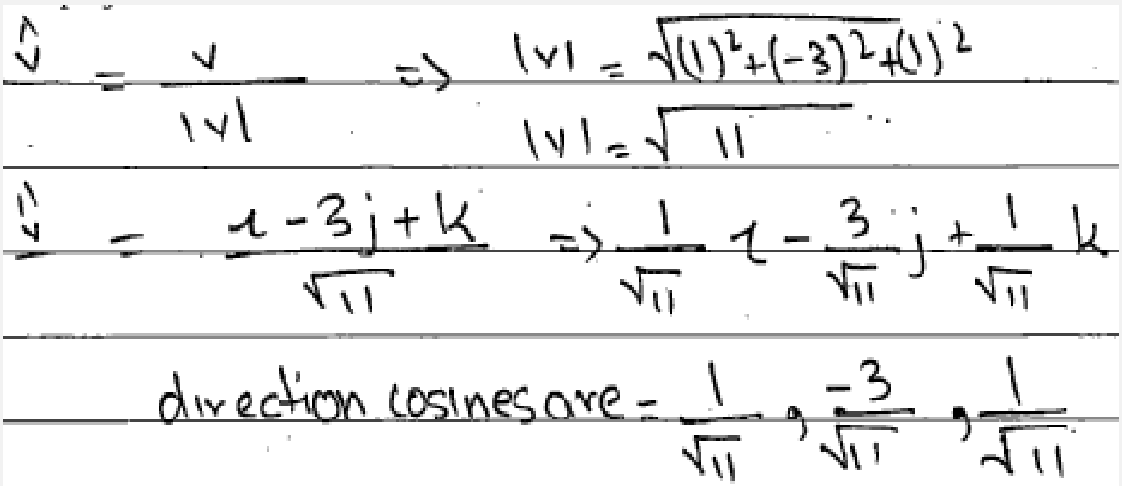
Any Additional Suggestion:
Following are some interactive teaching strategies for teaching translation and rotation.
Hands-On Manipulatives: Provide students with physical objects like blocks or geometric shapes. Let them physically translate (slide) and rotate (turn) these objects to understand the concepts intuitively.

Whiteboard Demonstrations: Use a whiteboard or a digital whiteboard and demonstrate translations and rotations using drawings. Encourage students to participate by drawing their own translations and rotations.
Peer Teaching: Assign pairs or small groups of students to teach each other about translation and rotation. Encourage them to come up with their own explanations and examples.
Real-Life Examples: Show how translation and rotation are used in real life. For example, you can discuss how these concepts are used in architecture, design, robotics, or video game development.
Interactive Software Tools: Use interactive software tools like GeoGebra or Desmos to demonstrate translation and rotation concepts. These tools often allow students to manipulate objects and see how they change

Question No. 9a

Candidates were given the choice to attempt any ONE out of the two questions: 9a and 9b.

Question Text	Find the direction cosines of the vector $v = i - 3j + k$.
SLO No.	24.3.7
SLO Text	Find direction cosines of a vector.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for calculating the magnitude of the vector. 1 mark for finding the unit vector in the direction of the given vector. 1 mark for extracting the direction cosines.
Overall Performance	Most candidates attempted this question successfully found all the direction cosines for a given vector, demonstrating a clear understanding of the concept. However, a subset of the candidates only partially solved the question.
Description of Better Responses	Better responses showed that candidates calculated the cosines of all the three angles separately by applying the related formula. Firstly, they calculated the magnitude of the given vector and divided the coefficients of *i*, *j* and *k* vectors. Hence, the candidates could identify the cosines from the result.
Images of Better Responses	**Image (i)** 

$\hat{v} = \frac{v}{|v|} \Rightarrow |v| = \sqrt{(1)^2+(-3)^2+(1)^2}$

$|v| = \sqrt{11}$

$\hat{v} = \frac{i-3j+k}{\sqrt{11}} \Rightarrow \frac{1}{\sqrt{11}}i - \frac{3}{\sqrt{11}}j + \frac{1}{\sqrt{11}}k$

direction cosines are $= \frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}$

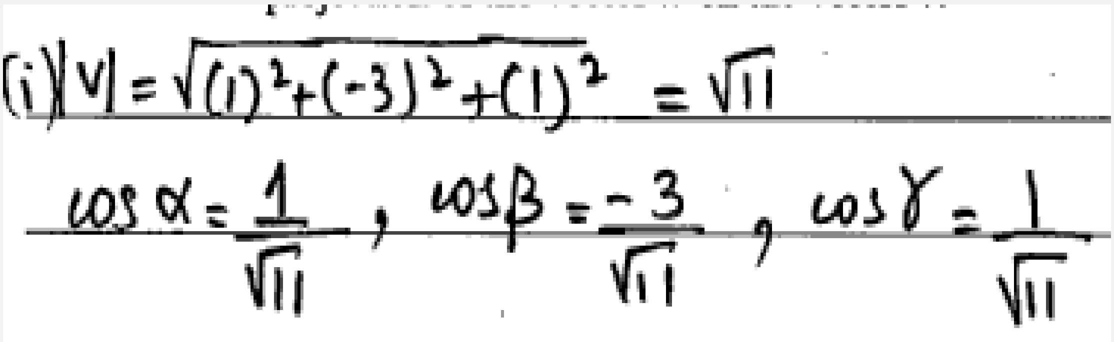
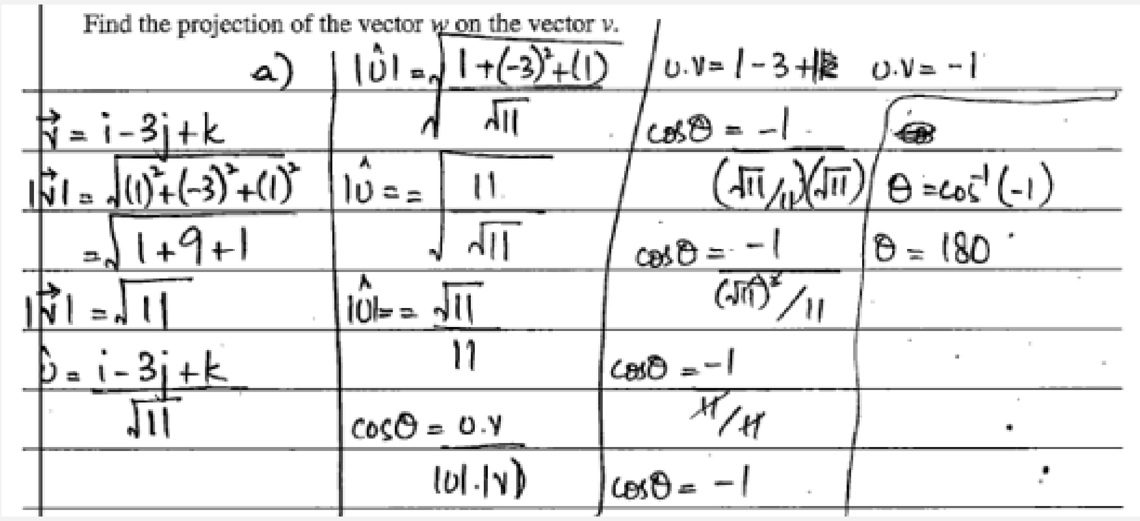
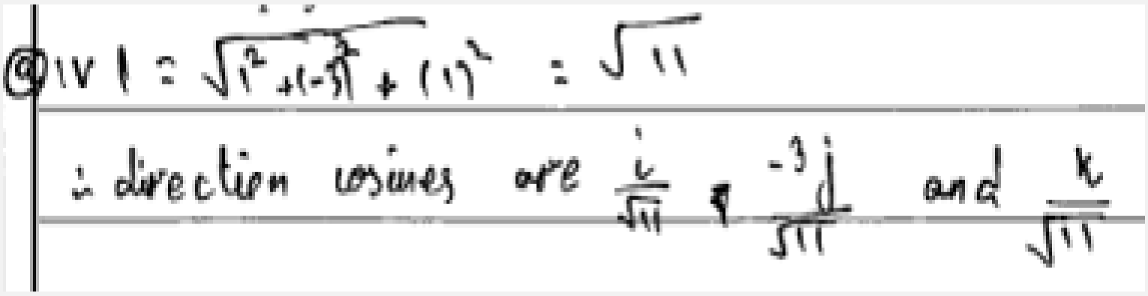
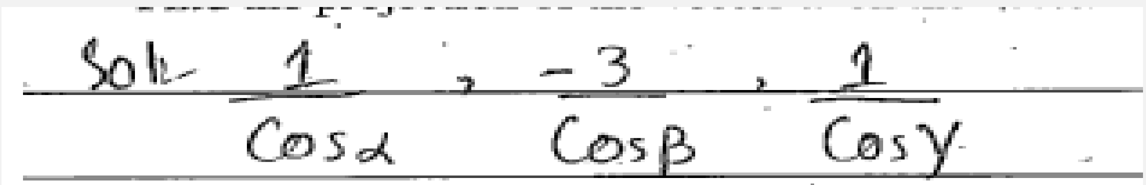
	Image (ii)  (i) $\lvert v \rvert = \sqrt{(1)^2+(-3)^2+(1)^2} = \sqrt{11}$ $\cos\alpha = \frac{1}{\sqrt{11}}$, $\cos\beta = \frac{-3}{\sqrt{11}}$, $\cos\gamma = \frac{1}{\sqrt{11}}$
Description of Weaker Responses	Weaker responses showed that some candidates did not have the concept of direction cosines. Some of candidates calculated wrong magnitude, while others calculated correct magnitude however, left in between after calculation of magnitude. Few of the examples are given below.
Images of weaker Response	**Image (i)**  Find the projection of the vector *w* on the vector *v*. a) $\vec{v} = i-3j+k$ $\lvert\vec{v}\rvert = \sqrt{(1)^2+(-3)^2+(1)^2}$ $= \sqrt{1+9+1}$ $\lvert\vec{v}\rvert = \sqrt{11}$ $\hat{v} = \frac{i-3j+k}{\sqrt{11}}$ $\lvert\hat{u}\rvert = \frac{\sqrt{1+(-3)^2+(1)}}{\sqrt{11}}$ $\lvert\hat{u}\rvert = \frac{\sqrt{11}}{\sqrt{11}}$ $\lvert\hat{u}\rvert = \frac{\sqrt{11}}{11}$ $\cos\theta = \frac{u.v}{\lvert u\rvert.\lvert v\rvert}$ $u.v = 1-3+1$ $u.v = -1$ $\cos\theta = \frac{-1}{(\sqrt{11}/_{11})(\sqrt{11})}$ $\cos\theta = \frac{-1}{(\sqrt{11})^2/11}$ $\cos\theta = \frac{-1}{11/11}$ $\cos\theta = -1$ $\theta = \cos^{-1}(-1)$ $\theta = 180$ **Image (ii)**  (a) $\lvert v \rvert = \sqrt{1^2+(-3)^2+(1)^2} = \sqrt{11}$ ∴ direction cosines are $\frac{i}{\sqrt{11}}$, $\frac{-3j}{\sqrt{11}}$ and $\frac{k}{\sqrt{11}}$ **Image (iii)**  Soln $\frac{1}{\cos\alpha}$, $\frac{-3}{\cos\beta}$, $\frac{1}{\cos\gamma}$

	Image (iv) We know that if a and ka have same sign then they are at same direction, if they have different sign they have different /opposite direction. $v = \hat{i} - 3\hat{j} + k$ it has opposite direction.

Suggestions for Improvement (Highlighted part)

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Any Additional Suggestion:
By implementing these teaching methodologies, candidates should have a better chance of improving their understanding and performance when it comes to solving problems related to direction cosines of vectors.
Visual Aids and Diagrams: Utilise visual aids and diagrams to help candidates visualise vector directions and their cosines. This can make abstract concepts more concrete.
Step-by-Step Approach: Teach a systematic step-by-step approach to finding direction cosines, emphasising the importance of each step in the process.
Real-Life Applications: Connect the concept of direction cosines to real-life applications or scenarios, showing why it's relevant and useful.
Problem Variations: Introduce variations of direction cosine problems, so candidates become adept at applying the concept to different scenarios.

Question No. 9b Candidates were given the choice to attempt any ONE out of the two questions: 9a and 9b.	
Question Text	Two vectors are defined as $w = 2i + 3j - k$ and $v = i + j + 2k$, having magnitudes $\sqrt{14}$ and $\sqrt{6}$ respectively.
SLO No.	24.3.10
SLO Text	Find the projection of a vector along another vector.
Max Marks	3
Cognitive Level	A
Checking Hints	1 mark for dot product 1 mark for cosine 1 mark for projection
Overall Performance	Most of the candidates primarily attempted this question, those who did engage with this question and addressed its requirements correctly were able to score full marks. However, it is noteworthy that a significant portion of the candidates who tackled this question only partially solved it, demonstrating gaps in their understanding of vector projection concepts. Among the weaker responses, some candidates struggled to exhibit any comprehension of vector projection, while others, although on the right track, made calculation errors that affected the accuracy of their solutions.
Description of Better Responses	Better responses showed all the necessary working and application of the formula. Also, in better responses, no calculation errors were found.
Images of Better Responses	**Image (i)**

w = magnitude = $\sqrt{(2)^2+(3)^2+(-1)^2} = \sqrt{14}$

$|w| = \frac{2}{\sqrt{14}} i + \frac{3}{\sqrt{14}} j - \frac{1}{\sqrt{14}} K$

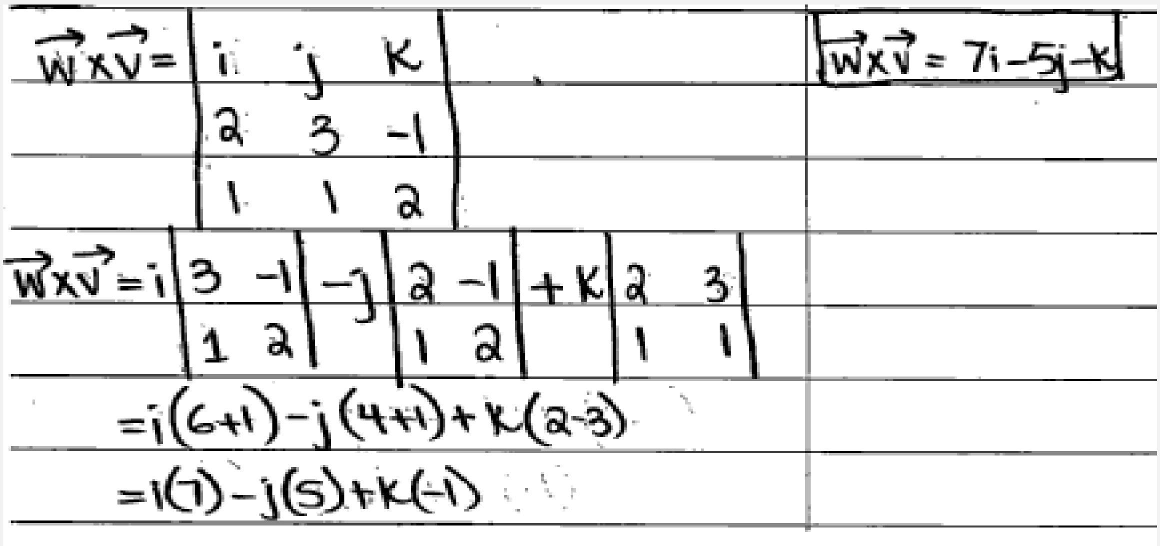
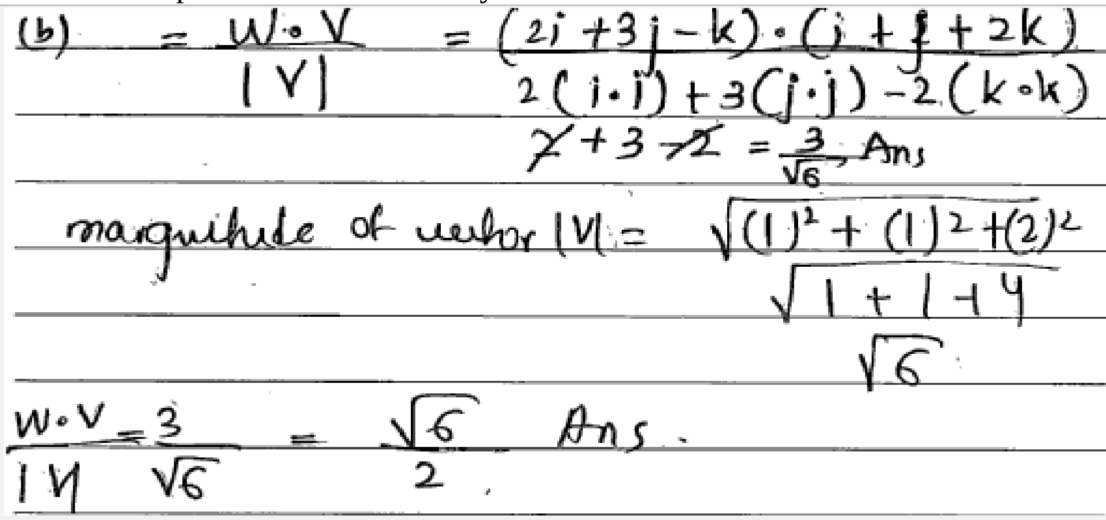
v = magnitude = $\sqrt{(1)^2+(1)^2+(2)^2} = \sqrt{6}$

$|w| = \sqrt{6}\left(\frac{2i}{\sqrt{14}} + \frac{3}{\sqrt{14}} j - \frac{1}{\sqrt{14}} k\right) = \frac{6\sqrt{2}}{\sqrt{14}} i + \frac{\sqrt{63}}{\sqrt{14}} j - \frac{\sqrt{6}}{\sqrt{14}} K$

$|v| = \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j + \frac{2}{\sqrt{6}} K$

$|v| = \sqrt{14}\left(\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}\right) = \frac{\sqrt{14}}{\sqrt{6}}\hat{i} + \frac{\sqrt{14}}{\sqrt{6}}\hat{j} + \frac{2\sqrt{14}}{\sqrt{6}}\hat{k}$

projection $= \frac{|w|\cdot|v|}{|v|} = \frac{\left(\frac{6\sqrt{2}}{\sqrt{14}}\hat{i} + \frac{\sqrt{63}}{\sqrt{14}}\hat{j} - \frac{\sqrt{6}}{\sqrt{14}}\hat{k}\right)\cdot\left(\frac{\sqrt{14}}{\sqrt{6}}\hat{i} + \frac{\sqrt{14}}{\sqrt{6}}\hat{j} + \frac{2\sqrt{14}}{\sqrt{6}}\hat{k}\right)}{\left(\frac{\sqrt{14}}{\sqrt{6}}\hat{i} + \frac{\sqrt{14}}{\sqrt{6}}\hat{j} + \frac{2\sqrt{14}}{\sqrt{6}}\hat{k}\right)} = \frac{6\sqrt{2}}{\sqrt{14}}\hat{i} + \frac{\sqrt{63}}{\sqrt{14}}\hat{j} \; \frac{\sqrt{6}}{\sqrt{14}}\hat{k}$ Ans

<table>
<tr><td></td><td>Image (ii)

$\vec{W}\times\vec{V} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 1 & 1 & 2\end{vmatrix}$
$\vec{W}\times\vec{V} = i\begin{vmatrix}3&-1\\1&2\end{vmatrix} - j\begin{vmatrix}2&-1\\1&2\end{vmatrix} + k\begin{vmatrix}2&3\\1&1\end{vmatrix}$
$= i(6+1) - j(4+1) + k(2-3)$
$= i(7) - j(5) + k(-1)$
$\boxed{\vec{W}\times\vec{V} = 7i-5j-k}$</td></tr>
<tr><td>Description of Weaker Responses</td><td>Lack of Understanding in Vector Projection:
This section highlights instances where candidates struggled to grasp the fundamental concepts related to vector projection, resulting in incomplete or incorrect responses.
Partial Correctness with Calculation Errors:
In this category, we examine responses that partially addressed vector projection questions correctly. However, these responses are marked down due to the presence of calculation errors that impact the overall accuracy of the solutions.</td></tr>
<tr><td>Image of weaker Response</td><td>
(b) $= \frac{W\cdot V}{|V|} = (2i+3j-k)\cdot(i+j+2k)$
$2(i\cdot i)+3(j\cdot j)-2(k\cdot k)$
$\cancel{2}+3\cancel{-2} = \frac{3}{\sqrt{6}}$ Ans
magnitude of vector $|V| = \sqrt{(1)^2+(1)^2+(2)^2}$
$\sqrt{1+1+4}$
$\sqrt{6}$
$\frac{W\cdot V}{|V|} = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$ Ans.</td></tr>
</table>

Suggestions for Improvement (Highlighted part)

How to Approach SLO	Pedagogy Used for that SLO	Assessment Strategies
• Understand the expectations of the command words • Look at the cognitive level • Identify the content that is required to answer that question (both in terms of understanding of concepts and any skills that may be required like analysing or evaluating) • Go through the past paper questions on that particular concept • Refer to the resource guide for extra resources	• Story Board • Cause and Effect • Fish and Bone • Concept Mapping • Audio Visual resources • Think, Pair and Share • Questioning Technique (Socratic approach) • Practical Demonstration	• Past paper questions • Discussion on E-Marking Notes • AKU-EB Digital Learning Solution powered by Knowledge Platform https://akueb.knowledgeplatform.com/login

Any Additional Suggestion:

By implementing these teaching strategies, candidates should have a better chance of improving their understanding and performance when it comes to solving problems related to vector projection along another vector. Additionally, addressing common errors and misconceptions will help candidates avoid making similar mistakes in the future.

Visual Representation: Use visual aids and diagrams to illustrate vector projections. Show how the projection relates to the angle between the vectors.

Geometric Interpretation: Explain the geometric interpretation of vector projection, emphasising that it represents the shadow of one vector on another.

Real-World Applications: Connect vector projection to real-world applications, such as physics, engineering, or computer graphics, to make it more relevant and engaging.

Interactive Learning: Use interactive tools or software to demonstrate vector projection in real-time, allowing candidates to manipulate vectors and observe the results.

Practice with Different Scenarios: Offer a variety of problems involving vector projection, including cases with different angles and magnitudes of vectors.

Error Analysis: Analyse common calculation errors made by candidates in the past and address these errors explicitly in the teaching materials.

Annexure A: Pedagogies Used for Teaching the SLOs

Pedagogy: Storyboard
Description: A visual pedagogy that uses a series of illustrated panels to present a narrative, encouraging creativity and critical thinking. It helps learners organise ideas, sequence events, and comprehend complex concepts through storytelling.
Example: In a Literature class, students are tasked with creating storyboards to visually retell a novel. They draw key scenes, write captions, and present their stories to the class, enhancing their reading comprehension and fostering their imagination.

Pedagogy: Cause and Effect
Description: This pedagogy explores the relationships between actions and consequences. By analysing cause-and-effect relationships, learners develop a deeper understanding of how events are interconnected and how one action can lead to various outcomes.
Example: In a History class, students study the causes and effects of the Industrial Revolution. They research and discuss how technological advancements in manufacturing led to significant societal changes, such as urbanisation and labour reform movements.

Pedagogy: Fish and Bone
Description: A method that breaks down complex topics into main ideas (the fish) and supporting details (the bones). This visual approach enhances comprehension by highlighting essential concepts and their relevant explanations.
Example: During a Biology class on human anatomy, the teacher uses the fish and bone technique to teach about the human skeletal system. Teacher presents the main components of the human skeleton (fish) and elaborates on each bone's structure and function (bones).

Pedagogy: Concept Mapping
Description: An effective way to visually represent relationships between ideas. Learners create diagrams connecting key concepts, aiding in understanding the overall structure of a subject and fostering retention.
Example: In a Psychology assignment, students use concept mapping to explore the various theories of personality. They interlink different theories, such as Freud's psychoanalysis, Jung's analytical psychology, and Bandura's social-cognitive theory, to see how they relate to each other.

Pedagogy: Audio Visual Resources
Description: Incorporating multimedia elements like videos, images, and audio into lessons. This approach caters to different learning styles, making educational content more engaging and memorable.
Example: In a General Science class, the teacher uses a documentary-style video to teach about the solar system. The video includes stunning visual animations of the planets, interviews with astronomers, and background music, enhancing students' interest and understanding of space.

Pedagogy: Think, Pair, and Share
Description: A collaborative learning technique where students ponder a question or problem individually, then discuss their thoughts in pairs or small groups before sharing with the entire class. It fosters active participation, communication skills, and diverse perspectives.
Example: In a Literature in English class, the teacher poses a thought-provoking question about a novel's moral dilemma. Students first reflect individually, then pair up to exchange their opinions, and finally participate in a lively class discussion to explore different viewpoints.

Pedagogy: Questioning Technique (Socratic Approach)
Description: Based on Socratic dialogue, this method stimulates critical thinking by posing thought-provoking questions. It encourages learners to explore ideas, justify their reasoning, and discover knowledge through a process of inquiry.
Example: In an Ethics class, the instructor uses the Socratic approach to lead a discussion on the meaning of justice. By asking a series of probing questions, the students engage in a deeper exploration of ethical principles and societal values.

Pedagogy: Practical Demonstration
Description: A hands-on approach where learners observe real-life applications of theories or skills. Practical demonstrations enhance comprehension, skill acquisition, and problem-solving abilities by bridging theoretical concepts with real-world scenarios.
Example: In a Food and Nutrition class, the instructor demonstrates the proper technique for filleting a fish. Students observe and then practice the skill themselves, learning the practical application of knife skills and culinary precision.

(**Note:** The examples provided in this annexure serve as illustrations of various pedagogies. It is important to understand that these pedagogies are versatile and can be applied across subjects in numerous ways. Feel free to adapt and explore these techniques creatively to enhance learning outcomes in your specific context.)

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