AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI

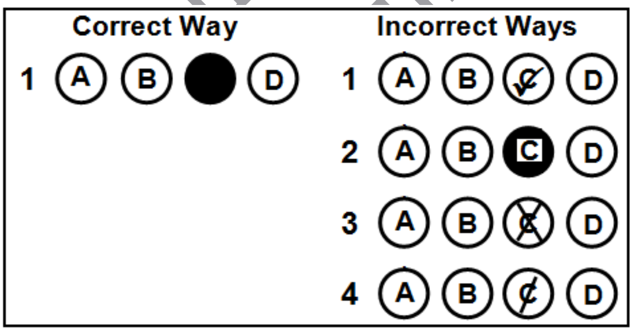
ANNUAL EXAMINATIONS (THEORY) 2023

Mathematics Paper I

Time: 1 hour 30 minutes Marks: 50

INSTRUCTIONS

1. Read each question carefully.
2. Answer the questions on the separate answer sheet provided. DO NOT write your answers on the question paper.
3. There are 100 answer numbers on the answer sheet. Use answer numbers 1 to 50 only.
4. In each question there are four choices A, B, C, D. Choose ONE. On the answer grid black out the circle for your choice with a pencil as shown below.



<u>Candidate's Signature</u>

5. If you want to change your answer, ERASE the first answer completely with a rubber, before blacking out a new circle.
6. DO NOT write anything in the answer grid. The computer only records what is in the circles.
7. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.
8. You may use a scientific calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Complex Numbers

$|z| = \sqrt{a^2 + b^2}$

Matrices and Determinants

$A_{ij} = (-1)^{i+j} M_{ij}$ $\quad AdjA = (A_{ij})^t$ $\quad A^{-1} = \frac{1}{|A|} AdjA$

Sequence & Series and Miscellaneous Series

$a_n = a_1 + (n-1)d$ $\quad A = \frac{a+b}{2}$ $\quad S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$a_n = a_1 r^{n-1}$ $\quad G = \pm\sqrt{ab}$ $\quad H = \frac{2ab}{a+b}$

$S_n = \frac{a_1(1-r^n)}{1-r}$ if $|r|<1$ $\quad S_n = \frac{a_1(r^n-1)}{r-1}$ if $|r|>1$ $\quad S_\infty = \frac{a_1}{1-r}$, where $|r|<1$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

Permutations, Combinations and Probability

${}^nP_r = \frac{n!}{(n-r)!}$ $\quad {}^nC_r = \frac{n!}{(n-r)!r!}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $\quad P(A|B) = \frac{P(A\cap B)}{P(B)}$ $\quad P(A\cap B) = P(A)\times P(B)$

Binomial Theorem and Mathematical Induction

$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \ldots + \binom{n}{n-1}a^1x^{n-1} + x^n$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots$$

$$T_{r+1} = \binom{n}{r}a^{n-r}x^r$$

Quadratic Equation

$x^2 - Sx + P = 0$ $\quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad D = b^2 - 4ac$

Introduction to Trigonometry and Trigonometric Identities

$l = r\theta$ $\quad \sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \mathrm{cosec}^2\theta$ $\quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\quad \tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\quad \sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\quad \tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ $\quad a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\quad \dfrac{a-b}{a+b} = \dfrac{\tan\dfrac{\alpha-\beta}{2}}{\tan\dfrac{\alpha+\beta}{2}}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$ $\quad \sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ $\quad \sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\quad \tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

Application of Trigonometry

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ac\sin\beta = \dfrac{1}{2}ab\sin\gamma$ $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$ $\quad R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$

$r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$ $\quad r = \dfrac{\Delta}{s}$ $\quad R = \dfrac{abc}{4\Delta}$

Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations

$\sin^{-1}A \pm \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} \pm B\sqrt{1-A^2}\right)$ $\quad \cos^{-1}A \pm \cos^{-1}B = \cos^{-1}\left(AB \mp \sqrt{(1-A^2)(1-B^2)}\right)$

$\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

1. The complex number $-3(2-i)+2(1+i)$ can be expressed in the form of $a+bi$ as

   A. $4-5i$.
   B. $4+5i$.
   C. $-4+5i$.
   D. $-4-5i$.

2. If $3z+ni=m$, then the complex conjugate of z will be

   (**Note**: z is a complex number.)

   A. $(m-ni)$.
   B. $\frac{1}{3}(m+ni)$.
   C. $(m+ni)$.
   D. $\frac{1}{3}(m-ni)$.

3. On complete factorisation of $9x^2+4y^2$, we get

   A. $(3x+i2y)(3x+i2y)$.
   B. $(3x-i2y)(3x-i2y)$.
   C. $(3x+i2y)(3x-i2y)$.
   D. $-(3x-i2y)(3x-i2y)$.

4. The additive inverse of a complex number $z+i$ is

   (**Note**: z is a complex number)

   A. $z-i$.
   B. $z+i$.
   C. $-z+i$.
   D. $-z-i$.

5. The multiplication of matrices $\begin{bmatrix} 2i & 3i \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ i & 0 \end{bmatrix}$ is

   (**Note:** $i=\sqrt{-1}$)

   A. $\begin{bmatrix} 2i+3 & 4i \end{bmatrix}$.
   B. $\begin{bmatrix} 2i-3 & 4i \end{bmatrix}$.
   C. $\begin{bmatrix} 8i & -3 \end{bmatrix}$.
   D. not possible.

6. If $A=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, then $|A|$ will be

A. -1
B. 0
C. 1
D. 2

7. $P=\begin{bmatrix}a & 0 & 0\\0 & c & 0\\0 & 0 & e\end{bmatrix}$ is a scalar matrix for non-zero real numbers *a*, *c* and *e* if,

A. $a=c=e$.
B. $a\neq c\neq e$.
C. $a=c\neq e$.
D. $a\neq c=e$.

8. The minor of the element *p* in the matrix $\begin{bmatrix}1 & 1 & -1\\1 & 2 & 0\\1 & p & 0\end{bmatrix}$ is

A. -2
B. -1
C. 1
D. 2

9. If the determinant of $\begin{bmatrix}a & b & c\\d & e & f\\g & h & i\end{bmatrix}$ is Δ, then the determinant of $\begin{bmatrix}3a & b & 3c\\3d & e & 3f\\3g & h & 3i\end{bmatrix}$ is

A. Δ.
B. 3Δ.
C. 9Δ.
D. 27Δ.

10. $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}^t \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}^t$ is equal to

A. $\left(\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)^t$.

B. $\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}\right)^t$.

C. $-\left(\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)^t$.

D. $-\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}\right)^t$.

11. In the given system of equations, the homogenous system of equation is

A. $x - y = 0,\ x + y = 5$
B. $x - 2 = 0,\ x + y = 0$
C. $x - y = 0,\ x + y = 0$
D. $x - y = 0,\ x - 5 = 0$

12. The adjoint of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is

A. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

C. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

D. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

13. If the n^{th} term of a sequence is $(-1)^{3n}(3n^2 - 3n - 20)$, then its 3^{rd} term will be

A. -2
B. -18
C. 2
D. 18

14. If the fourth term of an arithmetic sequence is 7 and the first term is 3, then the common difference will be

A. $-\frac{4}{3}$
B. -1
C. 1
D. $\frac{4}{3}$

15. The sum of the first eleven terms of an arithmetic series is 99. If the first term is 3, then the common difference will be

A. $\frac{12}{11}$
B. $\frac{4}{3}$
C. $\frac{6}{5}$
D. 1

16. For a given geometric sequence, if the first term is 2 and the common ratio $r = \frac{1}{2}$, then its 6^{th} term will be

A. 5
B. $\frac{9}{2}$
C. $\frac{1}{16}$
D. $\frac{1}{32}$

17. If T_n of an arithmetic sequence is $2n - 1$, then T_n of the associated harmonic sequence will be

A. $\frac{2}{n-1}$
B. $\frac{2}{n}-1$
C. $\frac{1}{2n-1}$
D. $\frac{1}{2n}-1$

18. If $2,\ p,\ \frac{1}{3}$ are in harmonic progression, then the value of $\left(\frac{1}{p}\right)^2$ will be

A. $\frac{9}{4}$
B. $\frac{9}{16}$
C. $\frac{49}{4}$
D. $\frac{49}{16}$

19. The sum of series $4\times1^3+4\times2^3+4\times3^3+4\times4^3+...+4\times10^3$ is

A. 3,025
B. 12,100
C. 24,200
D. 166,374

20. The sum of the first 202 natural numbers will be

A. 20,503
B. 20,200
C. 40,400
D. 41,006

21. If $(3-x)!=24$, then the value of x is equal to

A. -21
B. -19
C. -18
D. -1

22. The three events *K*, *L* and *M* are defined as $K=\{x:x+1=0\}$, $L=\{-1,1\}$ and $M=\{0,1\}$.

    Which of the given events are mutually exclusive?

    A. *K* and *M*
    B. *L* and *M*
    C. *K* and *L*
    D. *L* and *L*

23. In a sports festival, 6 students were lined up for a photograph. The number of possible arrangements of these students would have been

    A. 120
    B. 360
    C. 720
    D. 46,656

24. In a hockey training camp, 15 players participated.

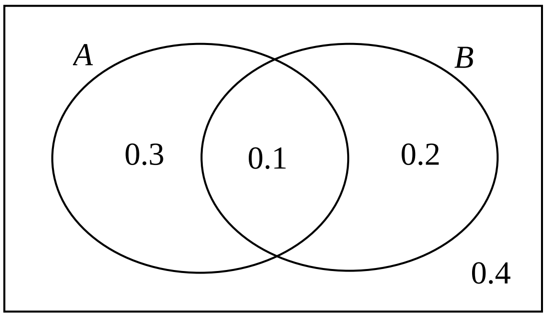
    If a team of 12 players is to be selected from these 15 players, then the possible number of team selections will be

    A. 455
    B. 1,365
    C. 479,001,600
    D. 217, 945,728,000

25. An urn contains 3 white balls, 5 green balls and 4 blue balls. If a ball is drawn from the urn at random, then the probability that this ball is white will be

    A. $\frac{1}{3}$
    B. $\frac{1}{12}$
    C. $\frac{3}{60}$
    D. $\frac{3}{12}$

26. Consider the given Venn diagram representing the probabilities of events *A* and *B*.



The probability of $A \cup B$ is

A. 0.4
B. 0.5
C. 0.6
D. 0.7

27. To prove that the statement $1+4+7+\ldots\ldots+(3n-2)=\frac{n(3n-1)}{2}$ is true for (k + 1) terms, the term that should be added to both sides will be

A. $3k-1$
B. $3k+1$
C. $\frac{3k(k+1)}{2}$
D. $\frac{(k+1)(3k+2)}{2}$

28. In the expansion of $(ax^2+b)^{19}$, the middle term(s) will be

A. 9^{th} term only.
B. 11^{th} term only.
C. 9^{th} and 10^{th} terms.
D. 10^{th} and 11^{th} terms.

29. In the expansion of $(2-x)^{15}$, the third term can be expressed as

A. $\binom{15}{2}(2)^{13}x^2$
B. $\binom{15}{3}(2)^{12}x^3$
C. $-\binom{15}{2}(2)^{13}x^2$
D. $-\binom{15}{3}(2)^{12}x^3$

30. Which of the following conditions is valid for the convergence of $(1-4x)^{\frac{1}{2}}$?

A. $|x| < 4$
B. $|x| > 4$
C. $|x| < \frac{1}{4}$
D. $|x| > \frac{1}{4}$

31. The equation $\frac{3}{2}x^2 - 6 = 3x$ can be converted into the standard quadratic form as

A. $x^2 - 2x + 4 = 0$
B. $x^2 - 2x - 4 = 0$
C. $x^2 + 2x - 4 = 0$
D. $x^2 + 2x + 4 = 0$

32. The value of $\sqrt{\omega^6} - \frac{3}{\sqrt{\omega^6}}$ is equal to

A. -2
B. -3
C. 0
D. 3

33. The sum of roots of the quadratic equation $x + \frac{2}{x} - 3 = 0$ is

A. -3
B. -2
C. 2
D. 3

34. If the roots of a quadratic equation are -1 and $-\frac{1}{3}$, then the equation will be

A. $x^2 - 3x + 4 = 0$
B. $x^2 + 3x + 4 = 0$
C. $3x^2 - 4x + 1 = 0$
D. $3x^2 + 4x + 1 = 0$

35. If $x^3 - bx^2 + 2bx - 15$ is divided by $x - b$, then the remainder is 1. The values of b will be

A. ± 4
B. ± 8
C. $\pm 2\sqrt{2}$
D. $\pm \frac{\sqrt{15}}{2}$

36. If $\frac{y^2}{x^2} = k^2$ and $\frac{y^2}{k^2} = 1$, then the value of x^2 is equal to

(**Note**: k is a constant)

A. $-k^2$
B. k^2
C. 1
D. 0

37. If θ lies in the fourth quadrant, then the CORRECT option for θ is

A. $\sin(\theta + 2\pi) > 0$ and $\cos(\theta + 2\pi) > 0$
B. $\sin(\theta + 2\pi) > 0$ and $\cos(\theta + 2\pi) < 0$
C. $\sin(\theta + 2\pi) < 0$ and $\cos(\theta + 2\pi) > 0$
D. $\sin(\theta + 2\pi) < 0$ and $\cos(\theta + 2\pi) < 0$

38. $\frac{\sqrt{1 - \sin^2\theta}}{\cos\theta} \times \frac{\sin\theta}{\sqrt{1 - \cos^2\theta}}$ is equal to

A. 0
B. 1
C. $\tan^2\theta$
D. $\cot^2\theta$

39. $\tan(2\pi - \theta)$ is equal to

A. $-\tan\theta$.
B. $-\cot\theta$.
C. $\tan\theta$.
D. $\cot\theta$.

40. The expression $\sin\left(\frac{5\pi}{2} + x\right)$, in terms of single angle expression, is equal to

A. $-\sin x$.
B. $-\cos x$.
C. $\sin x$.
D. $\cos x$.

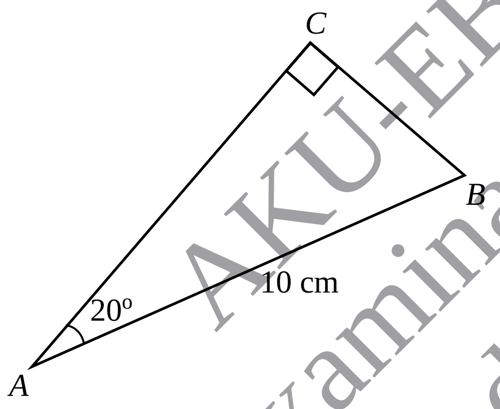
41. On applying trigonometric identities, the expression $\frac{\sin 2\alpha}{1-\sin^2\alpha}$ will be equal to

A. $2\cos\alpha$.
B. $2\sin\alpha$.
C. $2\tan\alpha$.
D. $2\cot\alpha$.

42. The product form of $\cos 5\theta - \cos 3\theta$ will be

A. $2\sin 4\theta \sin\theta$.
B. $2\cos 4\theta \cos\theta$.
C. $-2\cos 4\theta \cos\theta$.
D. $-2\sin 4\theta \sin\theta$.

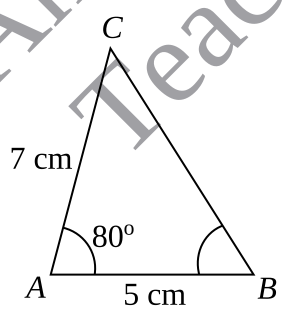
43. In the given triangle, the length of side *AC* is



NOT TO SCALE

A. 3.4 cm.
B. 3.6 cm.
C. 9.4 cm.
D. 9.6 cm.

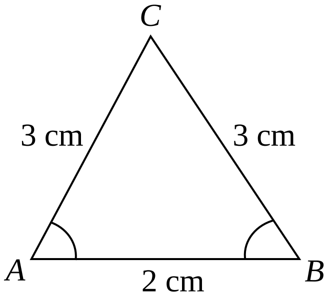
44. In the given triangle, the length of side *BC* is



NOT TO SCALE

A. 7.86 cm.
B. 8.24 cm.
C. 9.28 cm.
D. 8.94 cm.

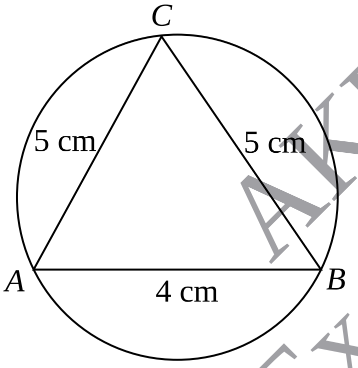
45. The area of the given triangle is



NOT TO SCALE

A. $\sqrt{2}\ \text{cm}^2$.
B. $2\sqrt{2}\ \text{cm}^2$.
C. $3\ \text{cm}^2$.
D. $4\ \text{cm}^2$.

46. If the area of the given triangle ABC is $2\sqrt{21}\ \text{cm}^2$, then the radius of the given circle will be



NOT TO SCALE

A. $\dfrac{2\sqrt{21}}{25}$ cm.
B. $\dfrac{25}{2\sqrt{21}}$ cm.
C. $\dfrac{\sqrt{21}}{50}$ cm.
D. $\dfrac{50}{\sqrt{21}}$ cm.

47. The domain of $y = 5\cos 10x$ is

A. $\{-\infty < x < \infty\}$.
B. $\{-5 \le x \le 5\}$.
C. $\{-10 < x < 10\}$.
D. $R - \{-10 \le x \le 10\}$.

48. The principal solution of $\sin\theta = -\frac{\sqrt{3}}{2}$ is

A. $\left\{\frac{\pi}{3}\right\}$.

B. $\left\{\frac{\pi}{3} + 2n\pi\right\}$.

C. $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$.

D. $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$.

49. The general solution of $\sqrt{2}\cos x = 1$ will be

A. $\left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$.

B. $\left\{\frac{\pi}{4} + 2n\pi\right\}$.

C. $\left\{\frac{\pi}{4} + n\pi\right\} \cup \left\{\frac{7\pi}{4} + n\pi\right\}$.

D. $\left\{\frac{\pi}{4} + 2n\pi\right\} \cup \left\{\frac{7\pi}{4} + 2n\pi\right\}$.

50. The trigonometric ratio $\left[-\sin(2A)\right]$ can also be expressed as

A. $-2\sin(-A)$.

B. $\sin(-2A)$.

C. $2\sin(-A)$.

D. $-\sin(-2A)$.

END OF PAPER

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