AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XII

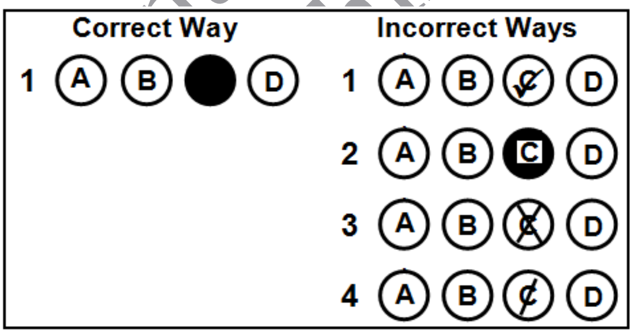
ANNUAL EXAMINATIONS (THEORY) 2023

Mathematics Paper I

Time: 1 hour 30 minutes Marks: 50

INSTRUCTIONS

1. Read each question carefully.
2. Answer the questions on the separate answer sheet provided. DO NOT write your answers on the question paper.
3. There are 100 answer numbers on the answer sheet. Use answer numbers 1 to 50 only.
4. In each question, there are four choices A, B, C, D. Choose ONE. On the answer grid, black out the circle for your choice with a pencil as shown below.



Candidate's Signature

5. If you want to change your answer, ERASE the first answer completely with a rubber, before blacking out a new circle.
6. DO NOT write anything in the answer grid. The computer only records what is in the circles.
7. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.
8. You may use a scientific calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Functions and Limits

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ $\quad \lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x}=\frac{1}{2\sqrt{a}}$ $\quad \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$ $\quad \lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x=e$ $\quad \lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$

Differentiation

$\frac{\Delta y}{\Delta x}=\frac{f(x+h)-f(x)}{h}$ $\quad \frac{d}{dx}(\sec x)=\sec x\tan x$ $\quad \frac{d}{dx}x^n=nx^{n-1}$ $\quad \frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$ $\quad \frac{d}{dx}(\tan x)=\sec^2 x$ $\quad \frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$ $\quad \frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cot x$

$\frac{d}{dx}\left[\sin^{-1}x\right]=\frac{1}{\sqrt{1-x^2}},\ x\in(-1,1)$ $\quad \frac{d}{dx}\left[\cos^{-1}x\right]=-\frac{1}{\sqrt{1-x^2}},\ x\in(-1,1)$ $\quad \frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{x^2+1},\ x\in R$ $\quad \frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{|x|\sqrt{x^2-1}},\ x\in[-1,1]'$

$\frac{d}{dx}\left[\operatorname{cosec}^{-1}x\right]=-\frac{1}{|x|\sqrt{x^2-1}},\ x\in[-1,1]'$ $\quad \frac{d}{dx}\left[\cot^{-1}x\right]=-\frac{1}{1+x^2},\ x\in R$ $\quad \frac{d}{dx}(a^x)=a^x\ln a$ $\quad \frac{d}{dx}(\log_a x)=\frac{1}{x}\times\frac{1}{\ln a}$

$\frac{d}{dx}(\sinh x)=\cosh x$ $\quad \frac{d}{dx}(\cosh x)=\sinh x$ $\quad \frac{d}{dx}(\tanh x)=\operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{cosech} x)=-\coth x\operatorname{cosech} x$ $\quad \frac{d}{dx}(\operatorname{sech} x)=-\tanh x\operatorname{sech} x$ $\quad \frac{d}{dx}(\coth x)=-\operatorname{cosech}^2 x$

$\frac{d}{dx}\left[f(x)\times g(x)\right]=f(x)\frac{d}{dx}\left[g(x)\right]+g(x)\frac{d}{dx}\left[f(x)\right]$ $\quad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}\left[f(x)\right]-f(x)\frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$

Maclaurin Series $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots+\frac{f^n(0)}{n!}x^n+\ldots$

Taylor's Series $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\frac{f^{iv}(a)}{4!}(x-a)^4+\ldots+\frac{f^n(a)}{n!}(x-a)^n+\ldots$

Integration

$\int f'(x)dx=f(x)+c$ $\quad \int[f(x)]^n f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+c,\quad (n\neq -1)$ $\quad \int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$

$\int\frac{f'(x)}{f(x)}dx=\ln f(x)+c$ $\quad \int\operatorname{cosec}x\,dx=\ln|\operatorname{cosec}x-\cot x|+c$ $\quad \int\sec x\,dx=\ln|\sec x+\tan x|+c$

Plane Analytical Geometry (Straight Line)

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$ $\quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ $\quad \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Point of internal division

$\left(\dfrac{k_1x_2+k_2x_1}{k_1+k_2}, \dfrac{k_1y_2+k_2y_1}{k_1+k_2}\right)$

Point of external division

$\left(\dfrac{k_1x_2-k_2x_1}{k_1-k_2}, \dfrac{k_1y_2-k_2y_1}{k_1-k_2}\right)$

$\dfrac{x-x_1}{\cos\alpha}=\dfrac{y-y_1}{\sin\alpha}=r$ (say)

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\dfrac{x}{a}+\dfrac{y}{b}=1$

$x\cos\alpha+y\sin\alpha=p$

$y=mx+c$

$y-y_1=m(x-x_1)$

$\theta=\tan^{-1}\left[\dfrac{m_2-m_1}{1+m_1m_2}\right]$

$\theta=\tan^{-1}\left[\dfrac{2\sqrt{h^2-ab}}{a+b}\right]$

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

$\Delta=\dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Circles

$(x-h)^2+(y-k)^2=r^2$

$x^2+y^2+2gx+2fy+c=0$

Equation of normal to a circle

$(y-y_1)(x_1+g)=(x-x_1)(y_1+f)$

Equation of tangent to a circle

$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

Length of tangent to a circle from a point (x_1, y_1), $l=\sqrt{x_1{}^2+y_1{}^2+2gx_1+2fy_1+c}$

Parabola

$x^2=4ay$

$(x-h)^2=4a(y-k)$

$y^2=4ax$

$(y-k)^2=4a(x-h)$

Ellipse

$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$, $a>b$

$b^2=a^2(1-e^2)$

$c=ae$

$\dfrac{(y-k)^2}{a^2}+\dfrac{(x-h)^2}{b^2}=1$, $a>b$

Hyperbola

$\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$

$b^2=a^2(e^2-1)$

$c=ae$

$\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$

Translation and Rotation

$x=X+h$, $y=Y+k$

$X=x\cos\theta+y\sin\theta$, $Y=y\cos\theta-x\sin\theta$

$\tan 2\theta=\dfrac{2h}{a-b}$

Vectors

$\underline{u}.\underline{v}=|\underline{u}||\underline{v}|\cos\theta$

Area of a triangle $=\dfrac{1}{2}|\underline{u}\times\underline{v}|$

$\underline{u}\times\underline{v}=\left(|\underline{u}||\underline{v}|\sin\theta\right)\hat{\underline{n}}$

$\underline{r}=\dfrac{q\underline{a}+p\underline{b}}{p+q}$

$\left|\overrightarrow{P_1P_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$\dfrac{1}{6}(\underline{u}\times\underline{v}).\underline{w}=\dfrac{1}{6}[\underline{u}\ \underline{v}\ \underline{w}]$

$\underline{u}.(\underline{v}\times\underline{w})=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Some Trigonometric Identities

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$

$\cos P+\cos Q=2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\cos P-\cos Q=-2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\sin P+\sin Q=2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin P-\sin Q=2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\sin 2\alpha=2\sin\alpha\cos\alpha$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$

1. The domain of the function $y = -|x-2|$ will be the set of

   A. real numbers
   B. real numbers except 2
   C. positive real numbers
   D. negative real numbers

2. The inverse function of $f(x) = 3 - \frac{x}{5}$ is

   A. $3 + 5x$
   B. $3 - 5x$
   C. $15 - 5x$
   D. $5x - 15$

3. The option that exemplifies explicit function is

   A. $y = e^{x+y}$
   B. $y = x + xy$
   C. $y = x + be^{x}$
   D. $2y + x + xy = 7$

4. Any number *y*, in the interval $[5,10)$ can be represented as

   A. $5 < y < 10$
   B. $5 \le y < 10$
   C. $5 \le y \le 10$
   D. $5 < y \le 10$

5. The derivative of $\frac{1}{\sqrt{x - a^2}}$, with respect to *x*, is

   A. $-a\left(x - a^2\right)^{-\frac{3}{2}}$
   B. $-a\left(x - a^2\right)^{-\frac{1}{2}}$
   C. $-\frac{1}{2}\left(x - a^2\right)^{-\frac{3}{2}}$
   D. $-\frac{1}{2}\left(x - a^2\right)^{-\frac{1}{2}}$

6. If $y = f(z)$ and $z = h(x)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{dx}{dy} \times \frac{dz}{dy}$.

B. $\frac{dy}{dz} \times \frac{dz}{dx}$.

C. $\frac{dz}{dx} \times \frac{dx}{dy}$.

D. $\frac{dz}{dy} \times \frac{dy}{dx}$.

7. If $t(x) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\frac{x}{3}$, then $\frac{dt}{dx}$ will be equal to

A. $-\frac{1}{\sqrt{9-x^2}}$.

B. $-\frac{3}{\sqrt{9-x^2}}$.

C. $\frac{1}{\sqrt{1-\left(\sqrt{3}\right)^2}} - \frac{3}{\sqrt{9-x^2}}$.

D. $\frac{1}{\sqrt{1-\left(\sqrt{3}\right)^2}} - \frac{1}{\sqrt{9-x^2}}$.

8. If $g(x) = a^{\ln(x+1)}$, then $g'(x)$ will be

A. $a^{\ln(x+1)} + \ln a$.

B. $a^{\ln(x+1)} \times \ln a$.

C. $\frac{a^{\ln(x+1)} \times \ln a}{x+1}$.

D. $\frac{a^{\ln(x+1)} + \ln a}{x+1}$.

9. If $y = \coth(\sin x)$, then the derivative of y with respect to x will be

A. $-\text{cosech}^2(\cos x)$.

B. $\text{cosech}^2(\cos x)$.

C. $-\text{cosech}^2(\sin x) \times \cos x$.

D. $\text{cosech}^2(\sin x) \times \cos x$.

10. If $f(t) = 1 + (1 - \cos t)^2$, then $f'(t)$ will be

A. $-2(\cos t - \cos 2t)$.
B. $-2\sin t \times (1 - \cos t)$.
C. $2(\cos t - \cos 2t)$.
D. $2\sin t \times (1 - \cos t)$.

11. For the curve $y = 2x^3 - 1$, the slope of normal at the point $(-1, -3)$ will be

A. -6
B. $-\frac{1}{6}$
C. $\frac{1}{6}$
D. 6

12. For $y = \sqrt{x}$, the second derivative $\frac{d^2y}{dx^2}$ is

A. $-\frac{1}{4x^{-\frac{3}{2}}}$.
B. $-\frac{1}{4x^{\frac{3}{2}}}$.
C. $\frac{1}{4x^{\frac{3}{2}}}$.
D. $\frac{1}{4x^{-\frac{3}{2}}}$.

13. The CORRECT form to resolve the rational expression $\frac{x-1}{(x+1)^2(x+2)}$ into partial fractions is

A. $\frac{A}{x+2} + \frac{B}{x+1}$.
B. $\frac{A}{x+2} + \frac{B}{(x+1)^2}$.
C. $\frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$.
D. $\frac{A}{x+2} + \frac{B}{x+1} + \frac{Cx+D}{(x+1)^2}$.

14. If $f'(x) = 3(x+2)^2$, then $f(x)$ will be

A. $\frac{3(x+2)^2}{2} + C.$

B. $3(x+2)^2 + C.$

C. $(x+2)^3 + C.$

D. $\frac{(x+2)^3}{3} + C.$

15. The integral $\int 9\,dx$ is equal to

A. $9x + C.$

B. $\frac{x}{9} + C.$

C. $9 + C.$

D. $9C.$

16. The general solution of the differential equation $\frac{dy}{dx} = x$ will be

A. $y = x + C.$

B. $y = x^2 + C.$

C. $y = \frac{x^2}{2} + C.$

D. $y = 2x^2 + C.$

17. Which of the following is/ are CORRECT about definite integrals?

I. It involves upper and lower limits of integration.
II. Definite integrals give a numerical value.
III. It involves a constant of integration.

A. I only
B. III only
C. I and II
D. II and III

18. The area under the graph of $f(x) = x$, between $x = -5$ and $x = 5$ is

A. 0 square units.
B. 20 square units.
C. 25 square units.
D. 50 square units.

19. The integral $\int_{5}^{5} dx$ is equal to

A. 0
B. 1
C. 10
D. 25

20. In the integral $\int \frac{\sin x}{1+\cos x} dx$, the useful substitution will be

A. $\cos x = y$.
B. $\sin x = y$.
C. $1+\cos x = y$.
D. $\frac{\sin x}{1+\cos x} = y$.

21. The equation of line passing through the points (a, a) and $(-a, -a)$ is

A. $x - y = 0$
B. $x + y = 0$
C. $ax + y = a$
D. $x + ay = -a$

22. If the point $\left(q, \frac{13}{3}\right)$ divides the line segment joining $(-1, -1)$ and $(7, 15)$ internally in the ratio 1:2, then the value of q will be

A. $\frac{5}{3}$.
B. $\frac{11}{3}$.
C. $\frac{17}{3}$.
D. $\frac{29}{3}$.

23. If the distance between two points $A(a, 1)$ and $B(2, 5)$ is $\sqrt{20}$, then the value of a will be

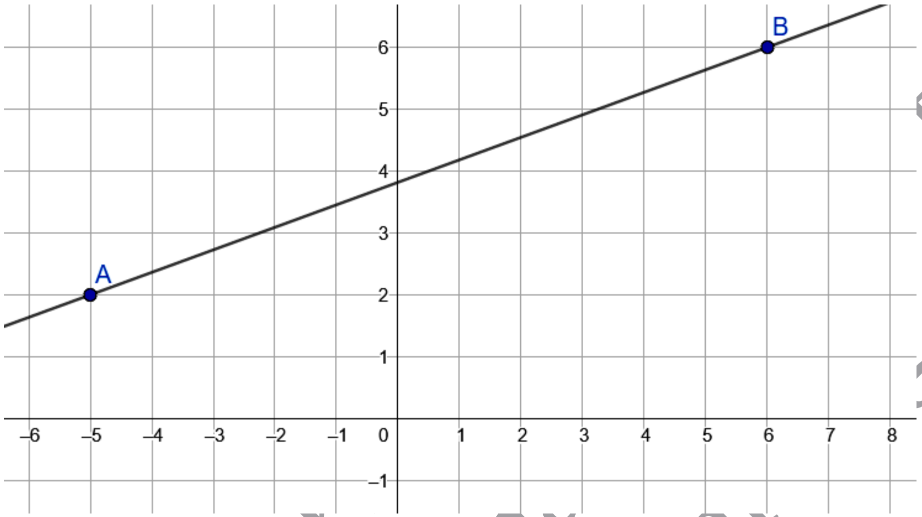
I. 20
II. 4
III. 0

A. I only.
B. III only.
C. I and II.
D. II and III.

24. Which of the following statements is FALSE about the medians of a triangle?

A. They are concurrent
B. They are of equal lengths
C. They meet at a single point
D. They intersect each other in the ratio of 1:2

25. The slope of the line *AB* in the given diagram is



A. $\frac{11}{4}$.
B. $\frac{4}{11}$.
C. $-\frac{5}{6}$.
D. $-\frac{6}{5}$.

26. The general form of the equation $y-a=\frac{1}{a}(x-a)$ is

A. $x-y-a+a^2=0$
B. $x-ay-a+a^2=0$
C. $x-y-a-a^2=0$
D. $x-ay-a-a^2=0$

27. The point-slope form of the equation $2x+3y-1=0$ will be

A. $y-\frac{1}{3}=-\frac{2}{3}(x-0)$.

B. $y-0=-\frac{2}{3}\left(x-\frac{1}{3}\right)$.

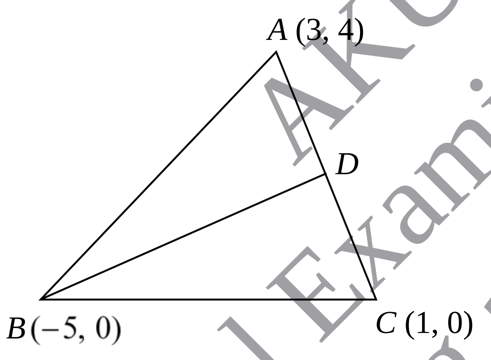
C. $y+\frac{1}{3}=\frac{2}{3}(x+0)$.

D. $y+0=\frac{2}{3}\left(x+\frac{1}{3}\right)$.

28. The angle between tangent and normal at point (0, 10) on the circle $x^2+y^2=100$ is

A. 0°
B. 30°
C. 60°
D. 90°

29. In the triangle *ABC*, *BD* is one of the altitudes.



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If the slope of *AC* is 2, then the equation for the altitude *BD* is

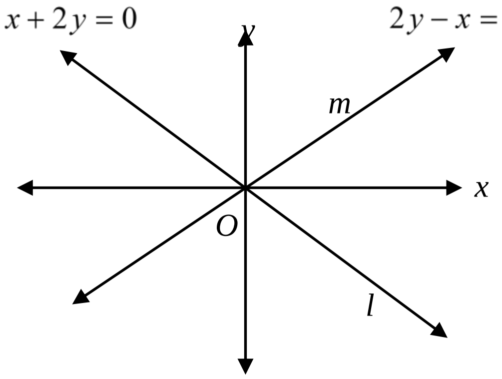
A. $y-0=-\frac{1}{2}(x+5)$.

B. $y+5=-\frac{1}{2}(x-0)$.

C. $y-0=\frac{2}{7}(x+5)$.

D. $y+5=\frac{2}{7}(x-0)$.

30. The joint equation of the given lines *l* and *m* is



A. $4y^2 - x^2 = 0$
B. $4y^2 + x^2 = 0$
C. $4y = 0$
D. $2x = 0$

31. For a linear programming problem, the optimal solution is defined as

A. the value of the decision variables giving extreme values of the function.
B. any value of the decision variables satisfying the constraints.
C. all corner points in the feasible region.
D. all points satisfying the constraints.

32. A cone is cut by a plane to form hyperbola.

If the angle between the axis of the cone and the plane cutting the cone is θ, then which of the following is TRUE?

A. $\theta = 0°$
B. $\theta = 90°$
C. $\theta > 90°$
D. $0 < \theta < 90°$

33. The equation of a circle passing though (5, 0) and (0, 5) will be

A. $x^2 + y^2 + 2x + 2y + 5 = 0$
B. $x^2 + y^2 - 2x - 2y - 5 = 0$
C. $x^2 + y^2 - 25 = 0$
D. $x^2 + y^2 - 5 = 0$

34. Which of the following lines is a tangent to the circle $x^2 + y^2 = 4$?

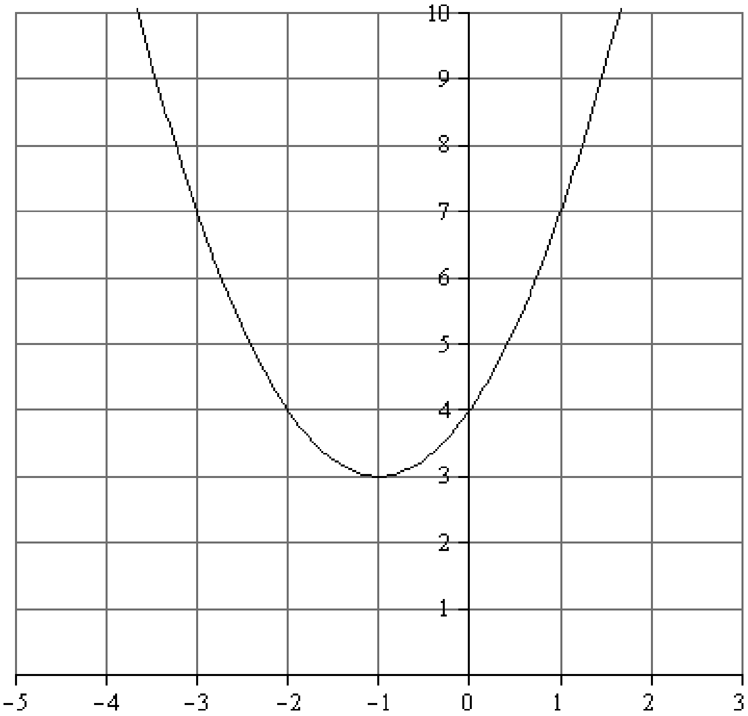
(**Note**: Condition for intersection is $a^2(1+m^2) - c^2 = 0$)

A. $y = x - 1$
B. $y = -3$
C. $y = 2$
D. $y = x + 2$

35. The equation of normal to the circle $x^2 + y^2 + 2x = 0$ at (− 1, 1) will be

A. $y - 1 = 0$
B. $y + 1 = 0$
C. $x - 1 = 0$
D. $x + 1 = 0$

36. Consider the given parabola.



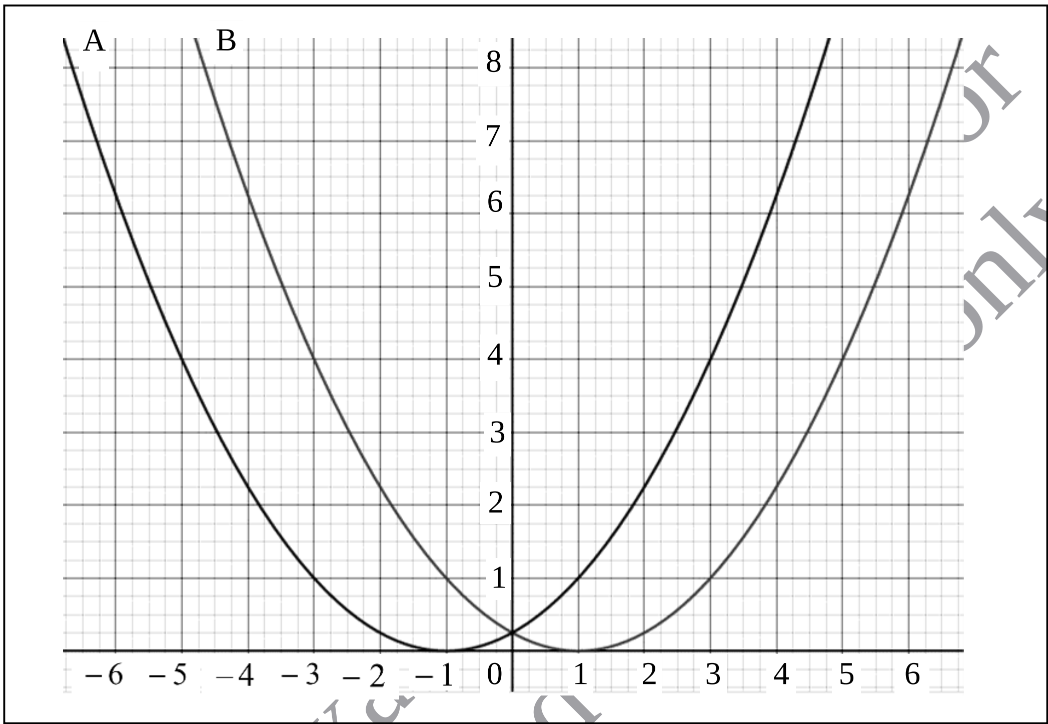
If $a = \frac{1}{4}$, then the equation of the parabola will be

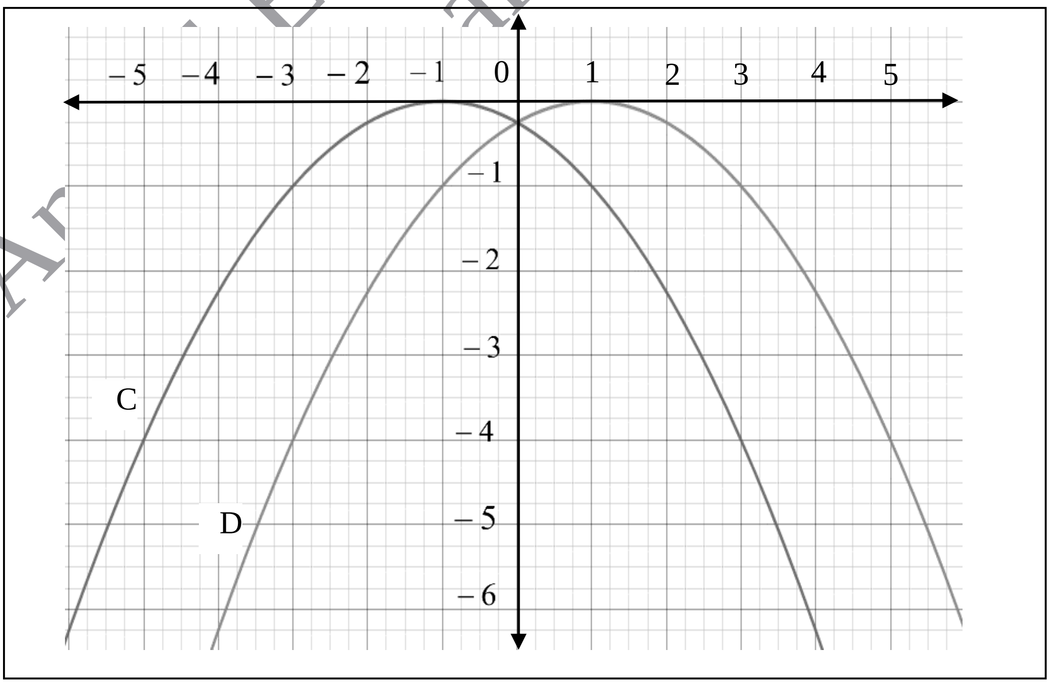
A. $y = x^2 - 2x - 4$
B. $y = x^2 + 2x + 4$
C. $x = y^2 - 2y - 4$
D. $x = y^2 + 2y + 4$

37. The elements of a parabola are tabulated as under.

Axis	**Vertex**	**Focus**	**Directrix**
$x-1=0$	(1, 0)	(1, 1)	$y=-1$

The graph of the parabola with the given information is





(**Note**: Options have been written along the graphs.)

38. If the axis of the parabola is $y=1$ and vertex is (0, 1) and opens right, then the equation of parabola will be

A. $y=x^2-2x+1$

B. $y=x^2+2x+1$

C. $x=y^2+2y+1$

D. $x=y^2-2y+1$

39. Which of the following is the value of eccentricity of an ellipse?

A. $\frac{2}{3}$

B. 1

C. 1.5

D. $\frac{5}{2}$

40. If the vertices of an ellipse are (3, −2) and (3, 8), then its centre will lie at the point

A. (3, 3).

B. (0, 5).

C. $(0,-5)$.

D. $\left(\frac{1}{2}, \frac{11}{2}\right)$.

41. The equation of tangent to the ellipse $\frac{x^2}{4}+y^2=1$ at (0, 1) is

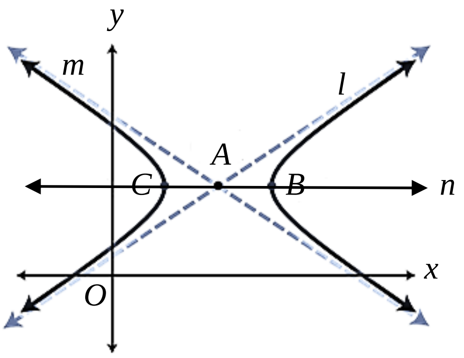
A. $y-1=0$

B. $x-1=0$

C. $x-4y+4=0$

D. $x+4y-4=0$

42. Observe the given hyperbola.



The elements of the given hyperbola are

	Asymptotes	Centre	Vertices
A	n and x	C	A and B
B	n and x	B	A and C
C	l and m	A	B and C
D	l and m	B	A and C

43. If $x^2 - y^2 = 64$, then the square of the eccentricity of the given hyperbola

A. is equal to 2
B. is less than 2
C. is greater than 2
D. cannot be determined

44. In *xy* Cartesian coordinate system, if axes remain parallel to the old axes and origin is shifted from *O* (0, 0) to $O'(-1, 0)$, then $2x + y - 9 = 0$ will be

(**Note**: (X, Y) are the coordinates of new axes.)

A. $2X + Y - 7 = 0$
B. $2X + Y - 8 = 0$
C. $2X + Y - 11 = 0$
D. $2X + Y - 10 = 0$

45. If a vector $\mathbf{i} - 2\mathbf{j} = n(-m\mathbf{i} + 2\mathbf{j})$, then the value of n

(**Note:** m and n are constants.)

A. is 0
B. is 1
C. is -1
D. cannot be determined

46. For the unit vector $\frac{5}{p}\boldsymbol{k}$, the value of p is equal to

A. 25
B. $\frac{1}{25}$
C. $\frac{1}{5}$
D. 5

47. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are two vectors, then projection of vector $\boldsymbol{a}$ along vector $\boldsymbol{b}$ will be

(**Note**: $\bullet$ represent dot product of vectors)

A. $\frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}|}$.
B. $\frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{b}|}$.
C. $\frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}|^2}$.
D. $\frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{b}|^2}$.

48. The scalar triple product of vectors $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$ and $\boldsymbol{c} = \boldsymbol{i} + 2\boldsymbol{k}$ is

A. -4
B. 0
C. 4
D. 6

49. The dot product of the vectors $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{j} - \boldsymbol{k}$ will be

A. -4
B. -3
C. 3
D. 4

50. If $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j}$ and $\boldsymbol{b} = -x\boldsymbol{i} - 3\boldsymbol{j}$ are two parallel vectors, then the value of x will be

A. -9
B. -3
C. 3
D. 9

END OF PAPER

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