AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XII

ANNUAL EXAMINATIONS (THEORY) 2023

Mathematics Paper II

Time: 1 hour 30 minutes Marks: 50

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. **Candidate's Signature**

RUBRIC

2. There are NINE questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.

6. You may use a scientific calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Functions and Limits

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1 \qquad \lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x}=\frac{1}{2\sqrt{a}} \qquad \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e \qquad \lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^x=e \qquad \lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$$

Differentiation

$$\frac{\Delta y}{\Delta x}=\frac{f(x+h)-f(x)}{h} \qquad \frac{d}{dx}(\sec x)=\sec x\tan x \qquad \frac{d}{dx}x^n=nx^{n-1} \qquad \frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x \qquad \frac{d}{dx}(\tan x)=\sec^2 x \qquad \frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x \qquad \frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cot x$$

$$\frac{d}{dx}\left[\sin^{-1}x\right]=\frac{1}{\sqrt{1-x^2}},\ x\in(-1,1) \qquad \frac{d}{dx}\left[\cos^{-1}x\right]=-\frac{1}{\sqrt{1-x^2}},\ x\in(-1,1) \qquad \frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{x^2+1},\ x\in R \qquad \frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{|x|\sqrt{x^2-1}},\ x\in[-1,1]'$$

$$\frac{d}{dx}\left[\operatorname{cosec}^{-1}x\right]=-\frac{1}{|x|\sqrt{x^2-1}},\ x\in[-1,1]' \qquad \frac{d}{dx}\left[\cot^{-1}x\right]=-\frac{1}{1+x^2},\ x\in R \qquad \frac{d}{dx}(a^x)=a^x\ln a \qquad \frac{d}{dx}(\log_a x)=\frac{1}{x}\times\frac{1}{\ln a}$$

$$\frac{d}{dx}(\sinh x)=\cosh x \qquad \frac{d}{dx}(\cosh x)=\sinh x \qquad \frac{d}{dx}(\tanh x)=\operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{cosech} x)=-\coth x\operatorname{cosech} x \qquad \frac{d}{dx}(\operatorname{sech} x)=-\tanh x\operatorname{sech} x \qquad \frac{d}{dx}(\coth x)=-\operatorname{cosech}^2 x$$

$$\frac{d}{dx}\left[f(x)\times g(x)\right]=f(x)\frac{d}{dx}\left[g(x)\right]+g(x)\frac{d}{dx}\left[f(x)\right] \qquad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}\left[f(x)\right]-f(x)\frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$$

Maclaurin Series $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots+\frac{f^n(0)}{n!}x^n+\ldots$

Taylor's Series $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\frac{f^{iv}(a)}{4!}(x-a)^4+\ldots+\frac{f^n(a)}{n!}(x-a)^n+\ldots$

Integration

$$\int f'(x)dx=f(x)+c \qquad \int[f(x)]^n f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+c,\quad (n\neq -1) \qquad \int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$$

$$\int\frac{f'(x)}{f(x)}dx=\ln f(x)+c \qquad \int \operatorname{cosec}x\,dx=\ln|\operatorname{cosec}x-\cot x|+c \qquad \int\sec x\,dx=\ln|\sec x+\tan x|+c$$

Plane Analytical Geometry (Straight Line)

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \qquad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1) \qquad \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Point of internal division

$$\left(\frac{k_1x_2+k_2x_1}{k_1+k_2}, \frac{k_1y_2+k_2y_1}{k_1+k_2}\right)$$

Point of external division

$$\left(\frac{k_1x_2-k_2x_1}{k_1-k_2}, \frac{k_1y_2-k_2y_1}{k_1-k_2}\right)$$

$$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}=r \text{ (say)}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$x\cos\alpha+y\sin\alpha=p$$

$$y=mx+c$$

$$y-y_1=m(x-x_1)$$

$$\theta=\tan^{-1}\left[\frac{m_2-m_1}{1+m_1m_2}\right]$$

$$\theta=\tan^{-1}\left[\frac{2\sqrt{h^2-ab}}{a+b}\right]$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Circles

$$(x-h)^2+(y-k)^2=r^2$$

$$x^2+y^2+2gx+2fy+c=0$$

Equation of normal to a circle

$$(y-y_1)(x_1+g)=(x-x_1)(y_1+f)$$

Equation of tangent to a circle

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

Length of tangent to a circle from a point (x_1, y_1), $l=\sqrt{x_1{}^2+y_1{}^2+2gx_1+2fy_1+c}$

Parabola

$$x^2=4ay \qquad (x-h)^2=4a(y-k) \qquad y^2=4ax \qquad (y-k)^2=4a(x-h)$$

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a>b \qquad b^2=a^2(1-e^2) \qquad c=ae \qquad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1,\ a>b$$

Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad b^2=a^2(e^2-1) \qquad c=ae \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Translation and Rotation

$$x=X+h,\ y=Y+k \qquad X=x\cos\theta+y\sin\theta,\ Y=y\cos\theta-x\sin\theta \qquad \tan 2\theta=\frac{2h}{a-b}$$

Vectors

$$\underline{u}.\underline{v}=|\underline{u}||\underline{v}|\cos\theta$$

Area of a triangle $=\frac{1}{2}|\underline{u}\times\underline{v}|$

$$\underline{u}\times\underline{v}=\left(|\underline{u}||\underline{v}|\sin\theta\right)\hat{\underline{n}}$$

$$\underline{r}=\frac{q\underline{a}+p\underline{b}}{p+q}$$

$$\left|\overrightarrow{P_1P_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$\frac{1}{6}(\underline{u}\times\underline{v}).\underline{w}=\frac{1}{6}[\underline{u}\ \underline{v}\ \underline{w}]$$

$$\underline{u}.(\underline{v}\times\underline{w})=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Some Trigonometric Identities

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos P+\cos Q=2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\cos P-\cos Q=-2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\sin P+\sin Q=2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P-\sin Q=2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

Q.1. (Total 3 Marks)

It is given that $f(x)=\frac{1}{(x-1)^2}$, $x \neq 1$ and $g(x)=x+1$ are two real valued functions.

For the given functions,

i. find $f \circ g$. (1 Mark)

ii. calculate $f \circ g(-1)$. (1 Mark)

iii. find $g^{-1}(x)$. (1 Mark)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

Q.2. (Total 10 Marks)

a.

i. Find the derivative of the function $f(x) = \left(1 - x^2\right)^{\frac{1}{2}}$ with respect to x by direct method. (2 Marks)

ii. For $y = (2x - 1)^2 \times (1 - e^x)$, find $\frac{dy}{dx}$. (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

b.

i. Find $\frac{dy}{du}$ for $y = 2x^2 - 1$ and $u = \frac{1-2x}{x}$. (3 Marks)

ii. For $y = \sin\sqrt{2x}$, find $\frac{dy}{dx}$. (2 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

c.

i. Find the equation of tangent to the curve $f(x) = x^2 - 4x + 5$ at $(1, -1)$. (3 Marks)

ii. Find the x-coordinate of the point on the curve $f(x) = 2x^2 - 4$ at which the tangent to $f(x)$ is parallel to the horizontal axis. (2 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

Q.3. (Total 10 Marks)

a.

i. Evaluate the integral $\int \tan^2 x dx$. (2 Marks)

ii. Integrate the function $y'(t) = \frac{\pi}{t^2}\sin\left(\frac{\pi}{t}\right)$ by substitution. (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

b. Evaluate the integral $\int (1+x)^2 \ln(1+x)\,dx$. (5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

c.

i. Calculate the area bounded by the function $y(t)=\cos\left(\frac{t}{2}\right)$ between $t=0$ to $t=\frac{\pi}{2}$.

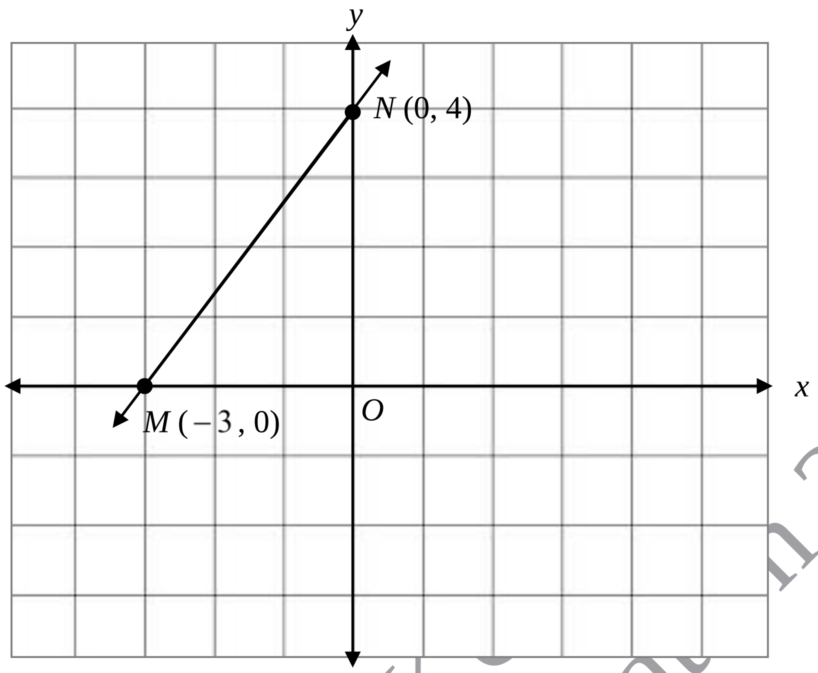
(2 Marks)

ii. Show that the general solution of the differential equation $\left(x^2-1\right)\frac{dy}{dx}+x\left(y+1\right)=0$ is $\ln\left(y+1\right)+\frac{1}{2}\ln\left(x^2-1\right)=C$.

(3 Marks)

Q.4. (Total 4 Marks)

In the given figure, the line passes through two points $M(-3, 0)$ and $N(0, 4)$.



Using the given figure,

i. find the slope of the given line. (2 Marks)

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ii. show that equation of the line is $4x - 3y + 12 = 0$. (2 Marks)

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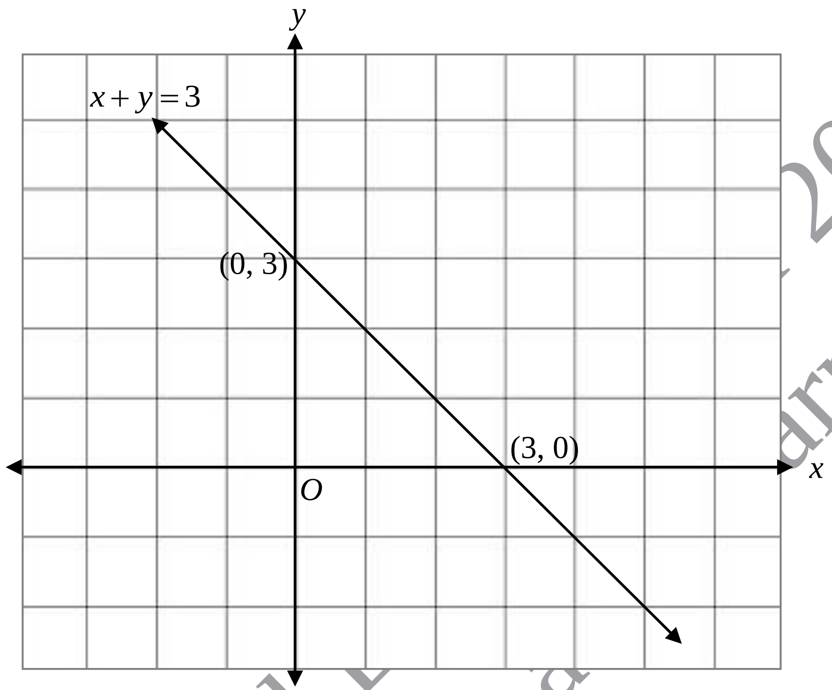
Q.5. (Total 4 Marks)

Using the given graph,

- draw the line $2x - y = 0$.
- find the maximum value of the function $f(x,\ y) = 2x + y$ subject to the following constraints.

$$2x - y \le 0$$
$$x + y \le 3$$
$$x \ge 0,\ y \ge 0$$



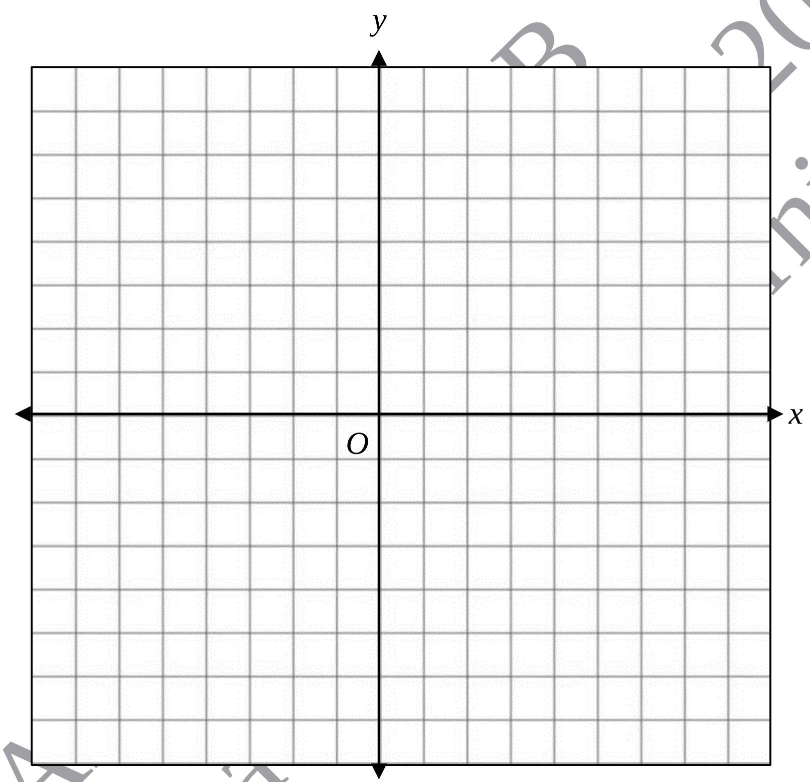
Q.6. (Total 4 Marks)

The equation of a circle is given by $3(x-3)^2 + 3y^2 = 27$.

For the given circle,

i. find the radius. (1 Mark)

ii. sketch the circle. (1 Mark)



iii. write the equation of the tangent to the circle at (3, 3). (2 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

Q.7. (Total 8 Marks)

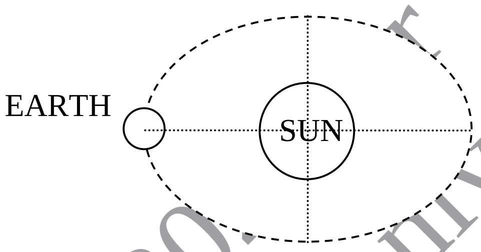
a. Find the equations of tangents to the parabola $y^2 = 8x$ at the points whose *x*-coordinate is 2.
(4 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

b. The Earth is moving around the Sun in an ELLIPTICAL path. During this motion, the shortest distance between their centres is 91 million miles while the farthest is 94.5 million miles.

If origin is considered at the centre of the Sun, then equation of this ellipse will have the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (4 Marks)

Find the



i. values of a and b. (1 Mark)

ii. eccentricity. (2 Marks)

iii. distance between directrices of the earth orbit. (1 Mark)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

c. Find the co-ordinates of foci, equations of directrices and lengths of the latera recta of the hyperbola $x^2 - y^2 = 9$. (4 Marks)

Q.8. (Total 4 Marks)

By transforming the equation $x^2+5y^2-2x+10y+5=0$ referred to a new origin and axes remaining parallel to the original axes, the first degree terms are removed. Find the coordinates of the new origin and the transformed equation.

(ATTEMPT ANY ONE PART FROM a AND b OF Q.9.)

Q.9. (Total 3 Marks)

a. Find the direction cosines of the vector $v = i - 3j + k$.

b. Two vectors are defined as $w = 2i + 3j - k$ and $v = i + j + 2k$, having magnitudes $\sqrt{14}$ and $\sqrt{6}$ respectively.

Find the projection of the vector *w* on the vector *v*.

END OF PAPER

Please use this page for rough work

Please use this page for rough work